08.128.140 Theoretische Physik 4: Statistische Physik
Theoretical Physics 4: Statistical Physics

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Homework set 3

Due November 13, 2023 by start of lecture.
Please note how long it took you to solve each problem.

3-1, 30 pts. *Binomial distribution, microstates and macrostates.* Consider a closed box of 100 coins that has been shaken vigorously and then opened. Assume each coin has a 50% chance of being heads and a 50% chance of being tails, and assume each coin is individually labeled. For each part, give at least an exact answer using factorials or exponential powers. *Aside: the situation where each coin is individually labeled as "heads" or "tails" denotes a microstate of the system, while the situation where the system as a whole has exactly "X" heads denotes a macrostate of the system. Each microstate is equally likely, while each macrostate is not. In particular, the combinatorial degeneracy of microstates into a given macrostate is responsible for the relative probability difference between different macrostates. Second aside: this is an example of a binomial distribution, since we are calculating probabilities where each individual event is characterized by one of two outcomes. In this case, the probability of each outcome is equally likely, but we can also have binomial distributions for outcomes that are not equally weighted.*

- A, 5 pts. How many microstates are available for the system?
- B, 5 pts. What is the *most probable* number of heads after the box is shaken?
- C, 5 pts. What is the number of microstates that correspond to part B? Divide your answer by the answer of part A to also give the probability associated with part B.
- D, 5 pts. Suppose we observed 40 coins showing heads and 60 coins showing tails. How many possible microstates would match this observation? What is the probability of this macrostate (before we open the box)?
- E, 10 pts. Suppose the coins had a 51% chance of showing heads and 49% chances of showing tails. Calculate the probability of finding 50 heads and 50 tails.

3-2, 20 pts. *Adiabatic index.* Assuming a monatomic ideal gas has an average kinetic energy per gas atom of $\langle E_{\text{kin}}\rangle = \frac{3}{2}k_B T$, we showed that the adiabatic index was $\gamma = 5/3$ such that $PV^\gamma = \text{const.}$ for a monatomic ideal gas.

- A, 10 pts. Calculate the adiabatic index for a diatomic ideal gas with $\langle E_{\text{kin}}\rangle = \frac{5}{2}k_B T$, where the diatomic molecules include rotational degrees of freedom.

B, 10 pts. Calculate the adiabatic index for a diatomic ideal gas with $\langle E_{\mathrm{kin}} \rangle = \frac{7}{2} k_B T$, where the diatomic molecules have both rotational and vibrational degrees of freedom.

3-3, 20 pts. *Universal efficiency.* Given the universal efficiency of a reversible (Carnot) engine $\eta(T_H, T_C)$,

$$\eta(T_H, T_C) \equiv \frac{T_H - T_C}{T_H} \; , \tag{1}$$

explain why negative temperatures are forbidden by the second law of thermodynamics.

3-4, 30 pts. *Engines.* Consider a heat engine: in one cycle, the heat engine draws 11 MJ of heat from a heat reservoir at 300 K, dumps 9.7 MJ of heat into a cold reservoir at 210 K, and performs work.

A, 5 pts. How much work energy is performed by the heat engine?

B, 5 pts. What is the efficiency of the heat engine?

C, 5 pts. What is the universal efficiency for the temperature source and sink? (Use the expression from problem 3-3.)

D, 5 pts. If the heat engine performs 550 cycles per second, how much power does the heat engine provide?

E, 10 pts. Now, we will make a refrigerator from the heat engine by running it in reverse, *i.e.* by swapping the input and output connections. If the engine efficiency is unchanged when running in reverse, and assuming the engine uses all of the work energy in part A, how much heat does the engine take from the cold source in one cycle, and how much heat is returned to the heat reservoir? Draw a diagram showing the net heat flow for the combined system of the heat engine and refrigerator. Explain why the combination of the original heat engine and the new refrigerator-as-heat-engine-in-reverse does not violate the second law of thermodynamics.