# 08.128.140 Theoretische Physik 4: Statistische Physik
# Theoretical Physics 4: Statistical Physics

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**Homework set 2**

**Due November 6, 2023 by start of lecture.**
**Please note how long it took you to solve each problem.**

2-1, 20 pts. *Prerequisite knowledge: basic statistics*

A, 10 pts. A thermostat for a refrigerator has measured the following temperatures in Celsius over a 12 hour period: 1.5, 1.8, 1.8, 2.1, 3.5, 2.3, 5.0, 3.8, 1.7, 1.9, 2.0, 1.6. Calculate the variance and standard deviation of the temperature readings.

B, 10 pts. Suppose a fair coin is flipped $10^{21}$ times. Using Stirling's formula, what is the order of magnitude (*i.e.* give your answer in powers of 10) for how many ways can you obtain exactly $4.9 \times 10^{20}$ heads (and correspondingly $5.1 \times 10^{20}$ tails)?

2-2, 20+5 pts. *Prerequisite knowledge: basic calculus.* You may find it useful to review Appendix C of Blundell and Blundell.

A, 10 pts. Calculate the expectation value of $|x|$ assuming a Gaussian distribution for $x$. Namely, calculate

$$E(|x|) = \int_{-\infty}^{\infty} |x| f_X(x) dx \tag{1}$$

for

$$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) , \tag{2}$$

where $\sigma$ and $\mu$ are real constants. You can use the fact that $\int f_X(x)dx = 1$ without proof. What is the expectation value of $x$ instead of $|x|$?

B, 10 pts. Calculate the expectation value of $x^2$ assuming the same Gaussian distribution as part A. *Hint: First perform a substitution of $y \equiv x - \mu$, then use integration by parts splitting up the $y^2$ prefactor of the exponential. You should put the exponential into the derivative term of the integration by parts identity.*

Bonus, 5 pts. Calculate the expectation value of $|x|^3$ assuming the same Gaussian distribution as part A. *Hint: Use a derivative approach on $\mu$.*

2-3, 24 pts. *Zeroth law of thermodynamics.* Recall that the zeroth law of thermodynamics states that equilibrium is a transitive property. Consequently, we can use the zeroth law to measure the temperature of unknown systems. First consider an ideal gas inside a piston with pressure $P_1 = 65$ kPa, $V = 30$ cm$^3$, and $N = 0.45A$, where $A$ is Avogadro's number. Suppose this piston is now placed in thermal contact (heat exchange is now allowed) with a second container filled with an unknown liquid, and you observe the pressure in the piston has dropped to 22 kPa.

A, 8 pts. What was the temperature of the gas inside the piston originally?

B, 8 pts. What is the temperature of the unknown liquid (assuming its temperature did not change in the entire process)?

C, 8 pts. If you disconnect the piston from the unknown liquid, to what volume should you compress the gas such that it returns to its original temperature?

2-4, 36 pts. *First law of thermodynamics.* Recall that the first law of thermodynamics states that the work performed on a sufficiently isolated system only depends on the beginning and ending points. This concept of "path independence" is also realized in classical mechanics and classical electrodynamics, and the breaking of the "path independence" principle is important for understanding friction and dissipative forces.

A, 12 pts. A ball is thrown into the air with an initial upward velocity of 16 m/s and lands on the ground 12 seconds later. Neglecting air friction, at what height should you drop an equivalent ball such that it lands on the group at the same speed as the first ball? When air friction is not ignored (but keeping your previous answer unchanged), which of the two balls will be faster when they hit the ground? (Assume terminal velocity has not been reached by either ball.) Sketch a cartoon of the two paths and isolate the "non-adiabatic" path that causes the difference in speed between the two balls.

B, 12 pts. An object with charge $477e$ moves upward in $\hat{z}$ through an electric field $\vec{E} = 19N/C\hat{x}$ (where $C$ is Coulombs) generated by two parallel capacitor plates that are separated by 2.3 m and have a vertical extent of 0.3 m. Sketch three different possible trajectories through the electric field that would result in the same net increase in kinetic energy for the object. If the object has a net positive horizontal displacement of 1.6 m from entering and exiting the electric field, calculate the work done by the field.

C, 12 pts. Calculate the work required to compress the piston as described in problem 2-3C. You will need to use the fact that an ideal gas is characterized by the constraint equation $PV^\gamma =$ constant for any trajectory that is adiabatic, where $\gamma > 1$ is the adiabatic index. This will allow you to integrate the work integral.