08.128.140 Theoretische Physik 4: Statistische Physik
Theoretical Physics 4: Statistical Physics

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Homework set 1

Due October 30, 2023 by start of lecture.
Please note how long it took you to solve each problem.

1-1, 16 pts. *Prerequisite knowledge: energy, work, heat, heat capacity*

A, 10 pts. A car with mass $m = 750$ kg is traveling at a speed of 120 km per hour. What is the kinetic energy of the car? Assuming the car begins at rest and all work is converted to kinetic energy with 13% efficiency, how much work is required from the engine? If the car accelerates at a constant rate from rest over a distance of 160 meters on a flat surface to its final speed of 120 km per hour, what is the force exerted by the engine? Assuming the engine uses gasoline as fuel with a heat of combustion value of 46 MJ per kg, how much gasoline was burned by the engine (you can neglect the change in the mass of the car during the acceleration process).

B, 6 pts. The specific heat capacity of water is 4.186 Joules per gram C (and one calorie is 4.186 Joules). A cup of 330 mL of room temperature water at 25 C is heated in a microwave to 80 C. How much energy went into the water? You can use the conversion that one liter of water has a mass of one kg (at standard pressure).

1-2, 19 pts. *Prerequisite knowledge: basic calculus*

A, 5 pts. Consider the function $f(x, y) = x^8 y^3 - \frac{x^5}{\sqrt{y}}$. Calculate $\partial f/\partial x$ and $\partial f/\partial y$, and demonstrate $\partial^2 f/(\partial x \partial y) = \partial^2 f/(\partial y \partial x)$.

B, 7 pts. Consider the function $g(f(x, y)) = f^2 - 7f$, with $f(x, y)$ defined in part A. Calculate the total derivative dg in terms of differentials dx and dy.

C, 7 pts. Consider the function $h(p) = \frac{1}{\sigma\sqrt{2\pi}} \exp[-p^2/(2\sigma^2)]$, with σ constant. Calculate dh/dp and d^2h/dp^2.

1-3, 30 pts. *Prerequisite knowledge: basic statistics.* Part of the complexity in statistical mechanics is the study of constrained systems, such as systems at a constant temperature or a constant pressure. Microscopically, a system with many degrees of freedom has many configurations that satisfy a macroscopic constraint. Therefore, it is important to know both combinatorial factors and how applying constraints change macroscopic properties of large sets.

A, 8 pts. Suppose you have a fair six-sided dice with each face labeled 1, 4, 5, 6, 17, and 200. What is the expected average of 100 rolls of the dice? If you rolled the dice twice, what is the probability that sum of the rolls is less than 20?

B, 8 pts. What is the mean of all integers between 1 and 100 (inclusive)? What is the mean of all prime numbers between 1 and 100 (inclusive)? What is the mean of all perfect square numbers between 1 and 100 (inclusive)?

C, 14 pts. *Stirling's formula.* Stirling's formula is a useful approximation for large factorials that appear as combinatorial factors of identical particles. The formula is

$$\ln n! \sim n \ln n - n \ . \tag{1}$$

Given an order of magnitude estimate for $10^{18}!$ and $10^{24}!$.

1-4, 35+5 pts. *Ideal gas law* The ideal gas law, $pV = Nk_BT = nRT$, where p = pressure, V = volume, N is the number of molecules, $k_B = 1.381 \times 10^{-23}$ Joules / K is Boltzmann's constant, n = the number of moles, $R = 8.314$ Joules / (mol K) is the ideal gas constant, and T is the temperature in Kelvin, is the equation of state for an ideal gas.

A, 7 pts. In the 2D coordinate space of p vs. V, sketch the shape of a set of isotherms.

B, 8 pts. What is the volume of an ideal gas with 10 moles given the pressure is 380 kPa and $T = 44$ Celsius?

C, 20 pts. Rewrite the ideal gas law to isolate k_B (or R) on one side of the equation. What is the required behavior for T if the volume is tripled, the number of particles is constant, and P is halved? Identify each pairwise combination of p, V, n, and T and their scaling relationship assuming the remaining two parameters are held constant. *Note: the three pairwise relationships of p, V and T have named laws after their discoverers: Boyle-Mariotte law for (p, V), Charles' law for (V, T), and Gay-Lussac's law for (p, T).*

Bonus, 5 pts. Based on the ideal gas law, write a few sentences explaining the importance of k_B being a nonzero numerical constant. In particular, how do you study the limit where $p \to 0$ and $V \to \infty$?