SUSY
Lecture 9.

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After our detour with SUSY phenomenology, the MSSM, and 2HDMs, we are transitioning to studies of more field theoretic aspects of SUSY theories.

This will encompass the Nelson-Seiberg theorem for spontaneous SUSY breaking, discussions of moduli fields & the SUSY vacuum, mediation scenarios for SUSY breaking, and ~~an~~ an extensive look at Super-QCD and Seiberg duality. We will also discuss independent topics such as the super-Higgs mechanism.

To begin with SUSY breaking, recall that the necessary & sufficient condition for SUSY was a non-zero vacuum energy. In non-SUSic theories, the scalar potential that determines the vacuum ~~is~~ generally prescribes a unique vacuum. A degeneracy in the vacuum arises when a global symmetry is spontaneously broken, as in the Mexican hat potential.

In SUSY theories, this is changed. The vacuum is not generally unique & the different points in field space that are degenerate vacua typically are not related by a symmetry transformation.

Consider Super-QED with a Fayet-Iliopoulos ξ term:

$$\mathcal{L} = \left(\frac{1}{4e^2}\int d^2\theta\, W^2 + h.c.\right) + \int d^4\theta \left(\bar{Q} e^{V} Q + \bar{\tilde{Q}} e^{-V} \tilde{Q} - \xi V\right)$$
$$+ \left(m \int d^2\theta\, Q\tilde{Q} + h.c.\right)$$

For the moment, set $m = \xi = 0$. Then the scalar potential is

$$V = \sum \frac{1}{2} D^2 = \frac{e^2}{2}\left(\bar{q}q - \bar{\tilde{q}}\tilde{q}\right)^2$$

Here, q + $\tilde{q}$ are the scalar components of Q and $\tilde{Q}$, and the bars denote complex conjugation.

Enforcing $V = \frac{e^2}{2}\left(\bar{q}q - \bar{\tilde{q}}\tilde{q}\right)^2 = 0$, we allow $q = \varphi$ + $\tilde{q} = \varphi$, where φ is a const. complex parameter.

So q + $\tilde{q}$ can acquire vevs but SUSY is unbroken.

This is called a "D-flat" direction, since the potential term is a D-term.

Now, super-Higgs: if $\varphi \neq 0$, then the super QED theory is Higgsed. The vector superfield V eats a chiral superfield: We can match the necessary dofs:

massless W_α: 2 dof for A_μ, 2 dof for gaugino $\tilde{\lambda}_\mu$,
chiral Q: 2 dof for q, 2 dof for χ_q
chiral $\tilde{Q}$: 2 dof for $\tilde{q}$, 2 dof for $\chi_{\tilde{q}}$.

After Higgsing,

massive V: 4 dof for $A_\mu + h$, 4 dof for Dirac gaugino $\tilde{\lambda}_\mu$
massless Φ: 2 dof for φ, 2 dof for χ_φ.

D-flatness is a generic feature for supersymmetric gauge theories, since $q = \varphi$, $\tilde{q} = \varphi \Rightarrow \sum_2 D^2 = 0$ is a vev for a combination of superfields with zero charge.

$$\Phi = \varphi(x_L) + \sqrt{2}\,\theta\,\psi(x_L) + \theta^2 F$$

is the chiral superfield from φ.

$\Phi^2 \sim Q\tilde{Q}$ is a chiral invariant + supergauge invariant.
Flat directions arise from analyzing all possible chiral invariants + the constraints between them. Vacuum manifolds (here, a $\mathbb{C}_1$ manifold) are parametrized by chiral invariants.

For $\xi = m = 0$, every point of the flat direction coincides with $Q\tilde{Q} = \Phi^2$. Φ is known a "moduli field".

Turn on $\xi \neq 0$, keep $m = 0$.

$$V = \frac{e^2}{2}\left(\bar{q}q - \bar{\tilde{q}}\tilde{q} - \xi\right)^2$$

Then
$$q = \sqrt{\xi}\, e^{i\alpha} \cosh\rho \qquad \Rightarrow \quad m_A^2 = 2e^2 \xi \cosh 2\rho$$
$$\tilde{q} = \sqrt{\xi}\, e^{i\alpha} \sinh\rho \qquad\qquad = 2e^2 \xi \sqrt{1+\bar{\varphi}\varphi}$$
$$Q\tilde{Q}\Big|_{\theta=0} = \frac{\xi}{2} e^{2i\alpha} \sinh 2\rho \equiv \frac{\xi}{2}\varphi$$

For $\Phi = \frac{2}{\xi} Q\tilde{Q}$.

$$\partial_\mu \varphi^\dagger \partial^\mu \varphi = 4(\cosh 2\rho)^2 \left[\partial_\mu \rho\, \partial^\mu \rho + (\tanh 2\rho)^2 (\partial_\mu \alpha\, \partial^\mu \alpha)\right]$$

Exercise: Photon field: $A_\mu = -\frac{i}{2} \frac{\left(\bar{q}\overleftrightarrow{\partial_\mu} q - \bar{\tilde{q}}\partial_\mu \tilde{q}\right)}{\bar{q}q + \bar{\tilde{q}}\tilde{q}} = \frac{\partial_\mu \alpha}{\cosh 2\rho}$

where we only keep dof in Φ light.

Boson kinetic term is then

$$\mathcal{L} = \frac{\xi}{4} \frac{1}{\sqrt{1+\bar{\varphi}\varphi}} \partial_\mu \bar{\varphi}\, \partial^\mu \varphi$$

The Kähler metric $G_{i\bar{j}} = \frac{\partial^2 K}{\partial\phi^i \partial\bar{\phi}^{\bar{j}}} = \frac{\xi}{4} \frac{1}{\sqrt{1+\bar{\varphi}\varphi}}$

+ Kähler potential is

$$K(\Phi, \bar{\Phi}) = \xi\left(\sqrt{1+\bar{\Phi}\Phi} - \operatorname{arctanh}\sqrt{1+\bar{\Phi}\Phi}\right)$$

We have calculated that the dynamics of the moduli field is a SUSic sigma model with the given Kähler potential.

Now, turn on $m \neq 0$ & put $\xi = 0$.

F-terms $\quad F_Q = -\bar{m}\bar{\tilde{q}}$

$\quad F_{\tilde{Q}} = -\bar{m}\bar{q}$

Force $q=0$, $\tilde{q}=0$. Higgs branch vanishes & vacuum is unique.

For $m \neq 0$, $\xi \neq 0$, we cannot generally satisfy D-flatness and F-flatness. SUSY is spontaneously broken. (Called D-term breaking, special for $U(1)$ theories). Also called Fayet-Iliopoulos mechanism.

We have now seen examples of F-term (O'Raifeartaigh model) and D-term spontaneous SUSY breaking.

Shifman Sec. 54

As you may expect, spontaneous breaking of a cont symmetry leads to massless dofs: Here spontaneous breaking of SUSY leads to massless Goldstinos. (fermionic Goldstones.)

Sketch proof: Consider vacuum corr. fcn. for supercurrent & fermion field. Div. of current is conserved & then action of fermion field by supercharges gives $\langle F \rangle$ exp. value, which is assumed nonzero. Then, must have pole at $p \to 0$, which corresponds to massless fermion.

It also turns ~~of~~ out that the fermion couples to the supercharges proportional to the square root of the vacuum energy density.

Thm: If no fermion can play the role of the massless Goldstino, SUSY cannot be spont. broken. (Ex. all chiral superfields massive at tree level). Must also match global quantum numbers b/t fermion + supercurrent.

Generally, this massless Goldstino is eaten when we consider local SUSY coupled to gravity. Then the gravitino becomes massive by eating the Goldstino + is very weakly coupled.