Supersymmetry
Lecture 8.
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More MSSM pheno + Digression about 2HDM.

Two Higgs Doublet Models.

Ref. Higgs Hunters Guide by Dawson, Gunion, Haber, Kane

Context: Recall MSSM introduces second Higgs scalar doublet.

$$H_u \sim (1, 2, 1/2)$$
$$H_d \sim (1, 2, -1/2)$$

Thus we have 2×4 scalar dofs.

3 are Goldstones that are eaten by $W^{+/-}$ + Z after EWSB.

The remaining 5 are: 3 neutral scalars, h^0, H^0, A^0
2 charged scalars: $H^{+/-}$

The most general scalar potential

$$\langle H_u \rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_u \end{pmatrix} \qquad \langle H_d \rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} v_d e^{i\xi} \\ 0 \end{pmatrix}$$

$$\begin{aligned} V = {} & \lambda_1 \left(|H_u|^2 - \frac{v_u^2}{2} \right)^2 + \lambda_2 \left(|H_d|^2 - \frac{v_d^2}{2} \right)^2 \\ & + \lambda_3 \left(\left(|H_u|^2 - \frac{v_u^2}{2}\right) + \left(|H_d|^2 - \frac{v_d^2}{2}\right) \right)^2 \\ & + \lambda_4 \left(|H_u|^2 |H_d|^2 - (H_u H_d)(H_d^\dagger H_u^\dagger) \right) \\ & + \lambda_5 \left(\mathrm{Re}[H_u H_d] - \frac{v_u v_d}{2} \cos\xi \right)^2 \\ & + \lambda_6 \left(\mathrm{Im}[H_u H_d] - \frac{v_u v_d}{2} \sin\xi \right)^2 \\ & + \lambda_7 \left(H_u H_d |H_d|^2 + c.c. \right) \end{aligned}$$

For $\lambda_5 = \lambda_6$, rotate away ξ.

λ_7 is $\mathbb{Z}_2$-breaking – need to impose $\mathbb{Z}_2$ for suppressing FCNCs.
Parentheses indicate $SU(2)$ contraction.

Origin of $\mathbb{Z}_2$ global symmetry:

SM Yukawas now have two Higgses to couple to:

$$\mathcal{L} = y_1 \bar{Q} \tilde{H}_u u_R + y_2 \bar{Q} H_u d_R + y_3 \bar{L} H_u e_R + y_4 \bar{Q} H_d u_R + y_5 \bar{Q} \tilde{H}_d d_R + y_6 \bar{L} \tilde{H}_d e_R + h.c.$$

All terms are allowed by gauge invariance.

In SM, Higgs field gets a vev ⇒ diagonalizing Yukawa matrix (by flavor symmetry), then mass basis is necessarily diagonal simultaneously with Higgs Yukawa interactions + Z interactions. [Only flavor-changing interaction is in the $W^{+/-}$ via CKM.]

In 2HDM w/o $\mathbb{Z}_2$, the Dirac masses arise from two separate $SU(2)\times U(1)$-breaking sources (v_u + v_d). Then, the flavor symmetry (which is not expanded) is used to diagonalize to the mass basis ⇒ This diagonalization procedure does not, in general, make ~~keep~~ the scalar h_u^0 + h_d^0 couplings flavor-diagonal as well. Fatal for 2HDM because on tree-level contributions to $K^0\overline{K^0}$ mixing + FCNC probes.

So, for pheno reasons, it's straightforward to assign $\mathbb{Z}_2$-parity such that each Dirac fermion only gets mass from one vev.

There are 4 such models:

Type 1: $y_4, y_5, y_6 = 0$. This is SM-like.

Variant: H_d has opposite $\mathbb{Z}_2$ from H_u + becomes <u>inert</u>.

Type 2: $y_2 = y_3 = y_4 = 0$, others non zero. This is MSSM (superpotential terms show up in h.c. from above)

Type X + Type Y: One Higgs couples to u_R + e_R, other for only d_R ; One Higgs couples to u_R + d_R, other for only e_R [lepton-specific].

- With one of these choices, λ_7 in the scalar potential is turned off, and the usual $U(3)$ flavor symmetries ensure that the Yukawa matrices for masses + couplings are diagonal in the mass basis.

Consider Type 2:

$$\mathcal{L} = y_1 \bar{Q} \tilde{H}_u u_R + y_5 \bar{Q} \tilde{H}_d d_R + y_6 \bar{L} \tilde{H}_d e_R + h.c.$$

(contract $SU(2)$ indices, assign $\langle H_u \rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_u \end{pmatrix}$ + $\langle H_d \rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} v_d \\ 0 \end{pmatrix}$)
(set $\xi = 0$)

$$= y_1 \bar{u}_L \frac{v_u}{\sqrt{2}} u_R + y_5 \bar{d}_L \frac{v_d}{\sqrt{2}} d_R + y_6 \bar{e}_L \frac{v_d}{\sqrt{2}} e_R + h.c.$$

Define $\tan\beta \equiv \frac{v_u}{v_d}$.

In general 2HDM models, the total vev must be matched to the known $W^{+/-}$ mass: SM $m_W^2 = \frac{g^2 v^2}{4}$ $(v = 246\text{ GeV})$

For 2HDM, H_u + H_d both contribute:

$$m_W^2 = \frac{g^2}{4}\left(v_u^2 + v_d^2\right), \text{ so } v_u^2 + v_d^2 = (246\text{ GeV})^2$$

(More precisely, v is measured from μ-decay lifetime, where g drops out.)

This v_u and v_d effectively rescale the SM Yukawa couplings:

$$m_t = y_t^{2HDM} \cdot \frac{v_u}{\sqrt{2}} = y_t^{2HDM} \cdot \frac{v \sin\beta}{\sqrt{2}} \Rightarrow y_t^{SM} = y_t^{2HDM} \sin\beta$$

$$m_b = y_b^{2HDM} \frac{v_d}{\sqrt{2}} = y_b^{2HDM} \frac{v \cos\beta}{\sqrt{2}} \Rightarrow y_b^{SM} = y_b^{2HDM} \cos\beta$$

For $\tan\beta$ very large or very small, the y_t^{2HDM} or y_b^{2HDM} coupling becomes non-perturbative in order to match the SM Yukawa.

From the scalar potential, λ_3 and λ_5 lead to off-diagonal terms in the mass squared matrix for the gauge eigenstates h_u^0 + h_d^0: for $\lambda_5 \neq \lambda_6$ and $\sin\xi \neq 0$, the pseudoscalar A^0 also mixes, making a 3×3 mass squared scalar mass matrix.

In the CP-conserving case, the h^0 + H^0 CP-even mass eigenstates arise by a mixing angle rotation α from h_u + h_d. It is normally assumed that the 125 GeV Higgs is h^0 + the rest of the scalar spectrum is heavier.

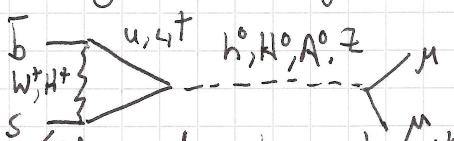
Because of the mixing α + parameter β, h^0 interacts with gauge bosons proportional to $\sin(\beta-\alpha)$ + H^0 interacts proportional to $\cos(\beta-\alpha)$. Note this still satisfies tree-level unitarity of gauge boson scattering (cf. Lee, Quigg, Thacker). Can look up all Feynman rules in Higgs Hunters.

From LHC, we must have $\cos^2(\beta-\alpha)$ small + $\sin^2(\beta-\alpha) \simeq 1$, by $h \to WW$ + $h \to ZZ$ signal strengths. Commonly achieved by decoupling limit or alignment limit (both make $\beta - \alpha \simeq \pi/2$). [Decoupling limit uses $m_{H^0} \gg v$, alignment uses parameter matching for finite m_H.]

Flavor probes of 2HDM.

In type-2 2HDM, the heavy Higgs bosons (H^0 + A^0) has $\tan\beta$-enhanced couplings to down-type quarks + leptons.

One of the most promising / constraining flavor channels is then $B_s^0 \to \mu^+\mu^-$.



Helicity-suppression in SM made less relevant in 2HDM.

Second process is $b \to X_s \gamma$, mainly probes charged Higgs.

Matching Type II 2HDM to MSSM:

In 2HDM, the quartic couplings are free parameters. But, as emphasized in the discussion about hard vs. soft SUSY breaking, dimensionless couplings must match a structure dictated by SUSY superpotential + gauge interactions.

Thus, recall that the scalar potential in MSSM given by
$$V = |F_i|^2 + \tfrac{1}{2}D^2$$
The $|F_i|^2$ will not give self-coupling quartics like $|H_u|^4$ + $|H_d|^4$ because the superpotential is linear in each chiral superfield.

The $\frac{1}{2}D^2$ gives the corresponding $\lambda_1 |H_u|^4$ + $\lambda_2 |H_d|^4$ from the 2HDM scalar potential earlier:

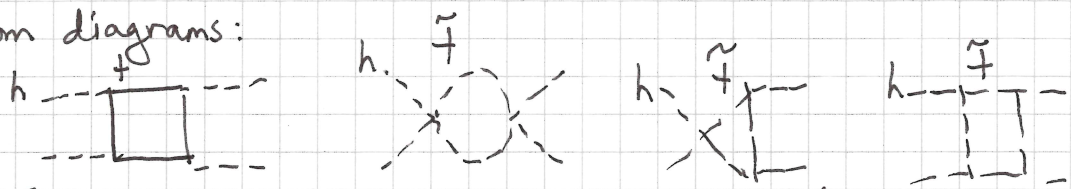
Calculating the H_u + H_d terms, we get
$$V \supset \frac{1}{8}(g^2+g'^2)\,|H_u|^4 + \frac{1}{8}(g^2+g'^2)\,|H_d|^4,$$
so the quartic couplings are given by gauge couplings. [This also ensures cancellation of quadratic divs. from EW gauge bosons in Higgs mass.] Note this contrasts with SM, where $V = -\mu^2|h|^2 + \lambda|h|^4$ gives two params $(\mu, \lambda) \leftrightarrow (v, m_h^2)$. In the MSSM, the tree level Higgs mass h^0 is bounded by m_Z:
$$m_h < m_Z\,|\cos 2\beta|$$
Thus, necessarily need large 1-loop correction from SUSY particles to make the 125 GeV result.

From Martin's review, the stop correction gives a log enhancement at 1-loop (should also go to two-loops from gluino) ⇒ in general, want to evaluate the 1-loop corrections for the scalar potential ⇐ Coleman-Weinberg calc.

$$m_h^2 = m_Z^2 \cos^2 2\beta + \frac{3}{4\pi} \sin^2\beta \, y_t^2 \Big[m_t^2 \ln\Big(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2} \Big) + c_{\tilde{t}}^2 s_{\tilde{t}}^2 \left(m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2 \right) \ln\Big(\frac{m_{\tilde{t}_2}^2}{m_{\tilde{t}_1}^2} \Big)$$

$$+ \frac{c_{\tilde{t}}^4 s_{\tilde{t}}^4}{m_t^2} \Big(\left(m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2 \right)^2 - \frac{1}{2} \left(m_{\tilde{t}_2}^4 - m_{\tilde{t}_1}^4 \right) \ln\Big(\frac{m_{\tilde{t}_2}^2}{m_{\tilde{t}_1}^2} \Big) \Big) \Big]$$

From diagrams:



These correct the quartic couplings & steepen the potential ⇒ increase the mass.

$c_{\tilde{t}}$ & $s_{\tilde{t}}$ are the c_θ & s_θ mixings for $\{\tilde{t}_L, \tilde{t}_R\} \xleftrightarrow[R_\theta]{} \{\tilde{t}_1, \tilde{t}_2\}$ rotation.

These come from diagonalizing

$$\mathcal{L} = -\left(\tilde{t}_L^* \; \tilde{t}_R^* \right) M_{\tilde{t}}^2 \begin{pmatrix} \tilde{t}_L \\ \tilde{t}_R \end{pmatrix}$$

$$M_{\tilde{t}}^2 = \begin{pmatrix} m_{Q_3}^2 + m_t^2 + \Delta_{\tilde{u}_L} & v(a_t^* s_\beta - \mu y_t c_\beta) \\ v(a_t s_\beta - \mu^* y_t c_\beta) & m_{u_3}^2 + m_t^2 + \Delta_{\tilde{u}_R} \end{pmatrix}$$

Mixing other up-type squarks is also present, depending on Yukawa structures & trilinear scalars A_t.

This structure of mixing also greatly determines MSSM DM pheno:

We have 4 neutralinos: $\tilde{B}$, $\tilde{W}^3$, $\tilde{H}_u^0$, $\tilde{H}_d^0$

$$\underbrace{\tilde{B}}_{\text{bino}},\ \underbrace{\tilde{W}^3}_{\text{wino}},\ \underbrace{\tilde{H}_u^0,\ \tilde{H}_d^0}_{\text{Higgsinos}}$$

Mass matrix (Majorana gauginos)

$$\mathcal{L} = -\frac{1}{2}(\psi_i^0)^T M_{\tilde{N}} \psi_j^0 + c.c.$$

$$M_{\tilde{N}} = \begin{pmatrix} M_1 & 0 & -c_\beta s_W m_Z & s_\beta s_W m_Z \\ 0 & M_2 & c_\beta c_W m_Z & -s_\beta c_W m_Z \\ -c_\beta s_W m_Z & c_\beta c_W m_Z & 0 & -\mu \\ s_\beta s_W m_Z & -s_\beta c_W m_Z & -\mu & 0 \end{pmatrix}$$

Symmetric, so only need one diagonalization.

Recall that in R_P-parity conservation, the lightest supersymmetric particle is stable $\Rightarrow$ for DM, needs to be $\tilde{\chi}_1^0$. From direct detection experiments, if the $\tilde{\chi}_1^0$ has too much gauge components from $\tilde{W}^3$, $\tilde{H}_u^0$, or $\tilde{H}_d^0$, then $\tilde{\chi}_1^0$ would interact too strongly with nucleons via (loops) $W^{+/-}$, Z + h exchange, so $\tilde{\chi}_1^0$ should be mostly $\tilde{B}^0$.

This has a separate problem of getting the correct relic density, since $\tilde{B}^0$ does not self-interact. But it is not difficult to adjust the components of $\tilde{\chi}_1^0$ to self-annihilate or to exercise coannihilation.