Lecture 5
Supersymmetry:
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Last time, we discussed gauge interactions, holomorphy, R-symmetry, and non-renormalization of the superpotential.

Today will discuss soft SUSY breaking ~~MSSM~~.
Dedicate 1-2 lectures to MSSM phenomenology.

Topics: ① MSSM collider pheno   Choose for next week.
② MSSM DM pheno
③ MSSM Higgs pheno
④ MSSM flavor pheno

SUSY breaking basics:

The necessary + sufficient condition for SUSY breaking is a non-vanishing vacuum energy.

We have already seen the chiral superfields give a scalar potential by $V(\phi, \phi^*) = |F_i|^2$, $F_i = \left.\frac{\partial W}{\partial \Phi_i}\right|_{\Phi_i = \phi_i}$

There's an additional contribution from gauge / real superfields: for super QED

$$\mathcal{L} = \left(\frac{1}{4e^2}\int d^2\theta\, W^2 + h.c.\right) + \int d^4\theta \left(\bar{Q} e^V Q + \bar{\tilde{Q}} e^{-V} \tilde{Q}\right)$$

$$+ \left(\text{superpotential}\right)$$

with $Q \sim +1$, $\tilde{Q} \sim -1$ charges.

p.2

Isolating the D-dependent terms in the Lagrangian, we have

$$\mathcal{L} = \left[\frac{1}{4e^2}\left(\int d^2\theta \; \theta^2\left(D^2 - \frac{1}{2}f^{\alpha\beta}f_{\alpha\beta}\right)\right) + \text{h.c.}\right]$$

$$+ \; D(\bar{q}q - \bar{\tilde{q}}\tilde{q}) \qquad \text{where } q \in Q,\ \tilde{q} \in \tilde{Q}, \text{ and } \bar{q} \equiv q^* . \quad \Big] \text{ scalar components}$$

Note we can also add an additional term:

$$\mathcal{L}_\xi = -\xi \int d^2\theta \, d^2\bar{\theta} \, V(x, \theta, \bar{\theta}) = -\xi D$$

This is only for Abelian gauge theories, known as the Fayet-Iliopoulos term.

Altogether, we have

$$\mathcal{L} = \frac{1}{2e^2} D^2 + D(\bar{q}q - \bar{\tilde{q}}\tilde{q}) - \xi D + \ldots$$

Then the scalar potential term arises from the D-term EOM:

$$\frac{\partial \mathcal{L}}{\partial D} = 0 \Rightarrow \frac{D}{e^2} + (\bar{q}q - \bar{\tilde{q}}\tilde{q}) - \xi = 0$$

$$D = -e^2(\bar{q}q - \bar{\tilde{q}}\tilde{q} - \xi)$$

Potential from D-terms is $V_D = \frac{1}{2e^2} D^2$.

In general,

$$V = |F_i|^2 + \frac{1}{2g_a^2}(D^a)^2,$$

$$F_i = \left.\frac{\partial W}{\partial \Phi_i}\right|_{\Phi_i = \phi_i}, \qquad D^a = \sum_i (g^a \phi_i^* T^a \phi_i)^*$$

So, for SUSY breaking, expectation value of one F or D must be non-zero.

Do not confuse nonzero field value with nonzero F.

Simple example:

Shifman 53.1, Dine 10.1

O'Raifeartaigh models:

Realize F-term breaking by writing superpotential s.t. not all chiral superfield F-terms can be simultaneously zero.

Shifman 53.2

(Contrast w/ D-term breaking)

Leads to spontaneous SUSY breaking.

Consider

$$W = \lambda A(X^2-\mu^2) + mBX$$

A, B, X are all singlet superfields.

$$\bar{F}_A = -\frac{\partial W}{\partial A} = \lambda(X^2-\mu^2)$$

$$\bar{F}_B = -\frac{\partial W}{\partial B} = mX$$

$$\bar{F}_X = -\frac{\partial W}{\partial X} = 2\lambda AX + mB.$$

Note $F_A=0$ and $F_B=0$ are incompatible.

The scalar potential is then, assume λ, m, μ real.

$$V = \lambda^2 |X^2-\mu^2|^2 + m^2|X|^2$$

Exercise: Minimize V.

Get $X=0$ or $X^2 = \mu^2 - \frac{m^2}{2\lambda^2}$

Evaluating, $V(X=0) = \lambda^2 |\mu|^4$

$$V\left(X=\mu^2-\frac{m^2}{2\lambda^2}\right) = \lambda^2\frac{m^4}{4\lambda^4} + m^2\mu^2 - \frac{m^4}{2\lambda^2}$$

Shifman typo?

$$= m^2\mu^2 - \frac{m^4}{4\lambda^2}$$

This is the lower minimum if $m^2 \geq 2\lambda^2\mu^2$

Assume $m^2 < 2\lambda^2\mu^2$, then live in vacuum with $X=0$. Then A is undetermined + $B=0$ from $\bar{F}_X = 0$. Will get a flat direction for shifting A, giving rise to massless Goldstino component (SUSY version of Goldstone's thm.)

Recall:

$$\mathcal{L} = \left(\int d^2\theta\, W + h.c.\right)$$

Dine, 10.1.1 Then $V = \sum_i \left|\frac{\partial W}{\partial \Phi_i}\right|^2_{\Phi_i = \phi_i}$, which gives boson masses

of $$m_{i\bar{j}}^2 = \frac{\partial^2 V}{\partial \phi_i \partial \phi_j^*} = \frac{\partial^2 W}{\partial \phi_i \partial \phi_k} \frac{\partial^2 W^*}{\partial \phi_k^* \partial \phi_j^*}$$

$$m_{ij}^2 = \frac{\partial^2 V}{\partial \phi_i \partial \phi_j} = \frac{\partial W^*}{\partial \phi_k^*} \frac{\partial^3 W}{\partial \phi_k \partial \phi_i \partial \phi_j}$$

We also have fermion masses

$$M_{F_{ij}} = \frac{\partial^2 W}{\partial \phi_i \partial \phi_j}$$

(logic, pick up $\theta^\alpha \theta^\beta$ from separate Φ_i fields, set any remaining superfields to scalar/F-term.)

From mBX, we get mass m for $\bar{\psi}_B \psi_B + \bar{\psi}_X \psi_X$.

Scalar masses have extra contribution from

$$V = \lambda^2 |X^2 - \mu^2|^2 = \lambda^2 |x^2 - \mu^2|^2$$

We get real fields $\frac{1}{\sqrt{2}}(Re(x) + i\, Im(x))$ with masses

$$m_{re}^2 = m^2 - 2\lambda^2\mu^2$$

$$m_{im}^2 = m^2 + 2\lambda^2\mu^2$$

Note this preserves a ~~super~~ <u>supertrace</u> condition

$$\mathrm{Str}\, M^2 = \sum_j (-1)^{2j} (2j+1)\, m_j^2 = 0$$

j = spin

The supertrace condition is generic for tree-level SUSY breaking by F-terms. It also is useful as a protection against quadratic corrections.

Next time: Soft SUSY breaking vs. hard SUSY breaking.