Supersymmetry

Lecture 3.

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Last time, we introduced the chiral and vector superfields and saw how SUSY transformations, namely the superderivatives, keep superfields as superfields.

Under the action of supertranslations, the bosonic and fermionic components of the superfields transformed into each other.

Recall chiral superfields:

$$\Phi(x_L, \theta) = \phi(x_L) + \sqrt{2}\,\theta^\alpha \psi_\alpha(x_L) + \theta^2 F(x_L)$$

$$= \phi(x) - i\,\theta\sigma^\mu\bar\theta\,\partial_\mu\phi - \frac{1}{4}\theta^2\bar\theta^2\partial^2\phi + \sqrt{2}\,\theta\psi + \frac{i}{\sqrt{2}}\theta^2\left(\psi^\alpha \overleftarrow{\partial}_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\right) + \theta^2 F$$

$$x_L^\mu = x^\mu - i\,\theta^\alpha (\sigma^\mu)_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}$$

$\partial_{\alpha\dot\alpha} = (\sigma^\mu)_{\alpha\dot\alpha}\partial_\mu$

$$\delta\phi = \sqrt{2}\,\epsilon^\alpha\psi_\alpha$$

$$\delta\psi_\alpha = -\sqrt{2}\,i\,\partial_{\alpha\dot\alpha}\phi\,\bar\epsilon^{\dot\alpha} + \sqrt{2}\,\epsilon_\alpha F$$

$$\delta F = i\sqrt{2}\,(\partial_{\alpha\dot\alpha}\psi^\alpha)\,\bar\epsilon^{\dot\alpha}$$

We see $\delta\phi \propto \psi$ and $\delta\psi \propto \partial\phi$.

Also, note δF transforms as total derivative.

SUSY blurs distinction between bosons and fermions, since SUSY transformation on component fields changes spin. Moreover, "natural" language of chiral superfields and vector superfields as basic building blocks in SUSY is an appealing feature since SM EW symmetry is chiral.

Given building blocks of chiral + vector superfields, how to construct interactions?

Recall Grassmanian integration

$$\int d\theta_i = 0 \qquad \int d\theta_i\, \theta_j = \delta_{ij}$$

We define $\int d^4\theta \equiv \int d^2\theta\, d^2\bar{\theta} = \int d\theta_1 d\theta_2 d\bar{\theta}_{\dot{1}} d\bar{\theta}_{\dot{2}}$

Normalization $\int \theta^2 \bar{\theta}^2 d^2\theta\, d^2\bar{\theta} = 1$

and $\int \theta^2 d^2\theta = 1, \quad \int \bar{\theta}^2 d^2\bar{\theta} = 1$

Dimensions $[\theta] = [\bar{\theta}] = [\text{length}]^{1/2} \qquad [d\theta] = [d\bar{\theta}] = [\text{Length}]^{-1/2}$

Recall $V(x^\mu, \theta, \bar{\theta}) \ni \theta^2 \bar{\theta}^2 D$

and $\delta D = \epsilon^\alpha (\partial_{\alpha\dot{\beta}} \bar{\lambda}^{\dot{\beta}}) + (\lambda^\beta \overleftarrow{\partial}_{\beta\dot{\alpha}}) \bar{\epsilon}^{\dot{\alpha}}$

which is a total derivative.

We identify $D = \int d^4\theta\, V$, and so the action $S = \int d^4x\, d^4\theta\, V(x, \theta, \bar{\theta})$ is superinvariant.

and the Lagrangian $\int d^4\theta\, V(x, \theta, \bar{\theta})$ is superinvariant up to a total derivative.

Given Φ and $\bar{\Phi}$ are chiral + antichiral fields, then $\bar{\Phi}\Phi$ is a vector superfield.

Shifman 49.2

$$\bar{\Phi}\Phi = \Big[\bar{\phi} + i\theta^\alpha(\partial_{\alpha\dot{\alpha}}\bar{\phi})\bar{\theta}^{\dot{\alpha}} - \frac{1}{4}\theta^2\bar{\theta}^2\partial^2\bar{\phi} + \sqrt{2}\,\bar{\theta}\bar{\psi} - \frac{1}{\sqrt{2}} i\bar{\theta}^2(\theta^\alpha \partial_{\alpha\dot{\alpha}}\bar{\psi}^{\dot{\alpha}}) + \bar{\theta}^2\bar{F}\Big]$$

$$\Big[\phi - i\theta^\alpha(\partial_{\alpha\dot{\alpha}}\phi)\bar{\theta}^{\dot{\alpha}} - \frac{1}{4}\theta^2\bar{\theta}^2\partial^2\phi + \sqrt{2}\,\theta\psi + \frac{1}{\sqrt{2}} i\theta^2(\psi^\alpha \overleftarrow{\partial}_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}) + \theta^2 F\Big]$$

$$= \bar\phi\phi + \sqrt{2}\,\theta\psi\bar\phi + \sqrt{2}\,\bar\theta\bar\psi\phi$$
$$+ i\theta^\alpha(\partial_{\alpha\dot\alpha}\bar\phi)\bar\theta^{\dot\alpha}\phi - i\theta^\alpha(\partial_{\alpha\dot\alpha}\phi)\bar\theta^{\dot\alpha}\bar\phi + 2(\theta\psi)(\bar\theta\bar\psi)$$
$$- \frac{i}{\sqrt{2}}\bar\theta^2\theta^\alpha(\phi\overleftrightarrow{\partial}_{\alpha\dot\alpha}\bar\psi^{\dot\alpha}) - \frac{i}{\sqrt{2}}\theta^2(\psi^\alpha\overleftrightarrow{\partial}_{\alpha\dot\alpha}\bar\phi)\bar\theta^{\dot\alpha}$$
$$+ \sqrt{2}\,\theta^2\bar\theta\bar\psi F + \sqrt{2}\,\bar\theta^2\theta\psi\bar F$$
$$+ \theta^2\bar\theta^2\left(\frac{1}{2}\partial_\mu\bar\phi\partial^\mu\phi - \frac{1}{4}\bar\phi\partial^2\phi - \frac{1}{4}\phi\partial^2\bar\phi + \frac{i}{2}\psi^\alpha\overleftrightarrow{\partial}_{\alpha\dot\alpha}\bar\psi^{\dot\alpha} + \bar F F\right)$$

Recall $\overleftrightarrow{\partial} \equiv \overrightarrow{\partial} - \overleftarrow{\partial}$

Then, $S_{kin} = \int d^4x\, d^4\theta\, \bar\Phi\Phi$

$\partial^{\dot\alpha\alpha} \equiv \partial_\mu(\bar\sigma^\mu)^{\dot\alpha\alpha}$

$$= \int d^4x \left(\partial_\mu\bar\phi\partial^\mu\phi + i\bar\psi_{\dot\alpha}\partial^{\dot\alpha\alpha}\psi_\alpha + \bar F F\right)$$

$$\boxed{\begin{aligned} &+\chi\sigma^\mu\bar\eta \\ &= -\bar\eta\bar\sigma^\mu\chi \end{aligned}}$$

after IBP. + discarding total derivatives.
We recognize this as the free kinetic term.

Aside: $\int d\theta_i = 0 \qquad \int d\theta_i\,\theta_j = \delta_{ij}$; $\int d(c\theta) = \int c^{-1} d\theta$ for c as #.

$\int d^4\theta \equiv \int d^2\theta\, d^2\bar\theta$

$\int \theta^2\bar\theta^2\, d^2\theta\, d^2\bar\theta = 1$ + $\int \theta^2 d^2\theta = 1$, $\int \bar\theta^2 d^2\bar\theta = 1$.

Everything else in integral vanishes because it does not saturate $\int d^4\theta = \int d^2\theta\, d^2\bar\theta$.

Now, the other way to construct Lagrangian terms from functions of (chiral) superfields is to integrate over $d^2\theta$. These functions are called the <u>superpotential</u>, which is generally a polynomial in chiral superfields.

We then get a superinvariant action by

$$S = \int d^4x \left(d^2\theta \; W(\Phi) + h.c. \right)$$

Since θ has mass dim. $-1/2$, $d\theta$ has mass dim. $+1/2$, so $W(\Phi)$ has mass dim. 3. Renormalizability then implies the highest power of superfields in $W(\Phi_i)$ is at most Φ_i^3.

Consider simplest "interaction": mass term.

$$W(\Phi) = \frac{m}{2} \Phi^2 ,$$

$$\Phi^2 = \phi^2 + 2\sqrt{2}\,\phi\,\theta^\alpha \psi_\alpha - \theta^2 \psi^2 + 2\theta^2 \phi F$$

Then $\mathcal{L} = \left(\int d^2\theta \; W + h.c. \right)$

$$= -\frac{m}{2}\psi^2 + m\phi F - \frac{m^*}{2}\bar{\psi}^2 + m^* \bar{\phi}\bar{F}$$

Now, analyze EOM for F terms.

$$\mathcal{L} = F\bar{F} + m\phi F + m^*\bar{\phi}\bar{F}$$

Non-dynamical $\Rightarrow \quad \bar{F} = -m\phi \, , \quad F = -m^*\bar{\phi}$

$$\text{So, } \mathcal{L} = -\frac{m}{2}\psi^2 - |m|^2|\phi|^2 - \frac{m^*}{2}\bar{\psi}^2 + \partial_\mu\bar{\phi}\,\partial^\mu\phi + i\bar{\psi}_{\dot{\alpha}}\,\bar{\partial}^{\dot{\alpha}\alpha}\psi_\alpha$$

Degenerate masses for scalar & spinors.

Since F terms are always non-dynamical, it is useful to identify their EOM. Kinetic part of action $\int d^4x\, d^4\theta\, \bar{\Phi}\Phi$ always gives $\bar{F}F$. The superpotential parts give

$$\mathcal{L}_F = \int d^2\theta \; W(\Phi) + \text{h.c.}$$

$$= F\left(\frac{\partial W(\Phi)}{\partial \Phi}\right)\Bigg|_{\Phi=\phi} + \bar{F}\left(\frac{\partial \bar{W}(\bar{\Phi})}{\partial \bar{\Phi}}\right)\Bigg|_{\bar{\Phi}=\phi^*}$$

Logic: $d^2\theta$ integral saturates the θ^2 superspace coordinates on $W(\Phi) = \sum a_n \Phi^n$, so can only have pure scalar component left.

Easy to take derivative of superpotential & replace superfield Φ by scalar component ϕ.

EOMs are
$$\bar{F} = -\left(\frac{\partial W(\Phi)}{\partial \Phi}\right)\Bigg|_{\Phi=\phi}$$
$$F = -\left(\frac{\partial \bar{W}(\bar{\Phi})}{\partial \bar{\Phi}}\right)\Bigg|_{\bar{\Phi}=\phi^*}$$

$\Rightarrow \bar{F}_i = -\left(\frac{\partial W(\Phi)}{\partial \Phi_i}\right)\Big|_{\Phi_i=\phi_i}$ for multiple chiral superfields

Then, scalar potential (critically important for studying SUSY breaking) is

$$V(\phi,\phi^*) = \left|\left[\frac{\partial W(\Phi)}{\partial \Phi}\Bigg|_{\Phi=\phi}\right]\right|^2 = |F|^2 \Rightarrow |F_i|^2$$

Exercise:

Consider Wess-Zumino model:

$$W = \frac{m}{2}\Phi^2 + \frac{\lambda}{3}\Phi^3$$

Determine scalar potential and Yukawa interactions.