Supersymmetry
Lecture 2.
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From last time, we saw that SUSY is unlike other symmetries you can consider in QFT. SUSY is essentially an extension of spacetime to superspace, where Poincaré algebra is extended to super-Poincaré algebra.

In superspace, we have $x^\mu, \theta_\alpha, \bar{\theta}_{\dot{\alpha}}$ as coordinates, with translation generators $p_\mu, Q_\alpha, \bar{Q}_{\dot{\alpha}}$, respectively.

To understand reps. of superspace + how supersymmetry meshes 4D-spacetime with fermionic $\theta_\alpha, \bar{\theta}_{\dot{\alpha}}$, consider

Group element of super-Poincaré algebra:

$$G(x^\mu, \theta, \bar{\theta}) = \exp[\, i(\theta Q + \bar{\theta}\bar{Q} - x^\mu p_\mu)]$$

If we act on another group element $G(a^\mu, \epsilon, \bar{\epsilon})$, we get

$$G(x^\mu, \theta, \bar{\theta})\, G(a^\mu, \epsilon, \bar{\epsilon})$$
$$= G(x^\mu + a^\mu + i\epsilon\sigma^\mu\bar{\theta} - i\theta\sigma^\mu\bar{\epsilon},\ \theta+\epsilon,\ \bar{\theta}+\bar{\epsilon})$$

which follows from the Baker-Campbell-Hausdorff lemma.

$$\exp(A)\exp(B) = \exp(A + B + \tfrac{1}{2}[A,B] + \cdots)$$

$\uparrow$ extra commutators like $[A,[A,B]]$, $[B,[A,B]]$.

Exercise! Ref. Shifman p. 422.

Identify $A = i(\theta Q + \bar{\theta}\bar{Q} - x^\mu p_\mu)$
$B = i(\epsilon Q + \bar{\epsilon}\bar{Q} - a^\mu p_\mu)$

Calculate commutator $[A,B]$.

Recall that $Q, \bar{Q}$ have the only non-trivial anti-commutator.

$$[\, i(\theta Q + \bar{\theta}\bar{Q} - x^\mu p_\mu),\ i(\epsilon Q + \bar{\epsilon}\bar{Q} - a^\mu p_\mu)]$$
$$= [-\theta Q, \epsilon Q + \bar{\epsilon}\bar{Q} - a^\mu p_\mu] + [-\bar{\theta}\bar{Q}, \epsilon Q + \bar{\epsilon}\bar{Q} - a^\mu p_\mu]$$
$$+\,0$$

$\uparrow$ p_μ commutes with everything.

p.2

Recall: $\{Q_\alpha, Q_\beta\} = \{\bar{Q}_{\dot\alpha}, \bar{Q}_{\dot\beta}\} = 0$

$[p_\mu, Q_\alpha] = [p_\mu, \bar{Q}^{\dot\alpha}] = 0$

$[p_\mu, p_\nu] = 0$

$\{Q_\alpha, \bar{Q}_{\dot\beta}\} = 2p_\mu (\sigma^\mu)_{\alpha\dot\beta}$

Also need $\{\theta^\alpha, \theta^\beta\} = \{\bar\theta^{\dot\alpha}, \bar\theta^{\dot\beta}\} = \{\theta^\alpha, \bar\theta^{\dot\beta}\} = 0$

$\left\{\frac{\partial}{\partial\theta^\alpha}, \frac{\partial}{\partial\theta^\beta}\right\} = \left\{\frac{\partial}{\partial\bar\theta^{\dot\alpha}}, \frac{\partial}{\partial\bar\theta^{\dot\beta}}\right\} = \left\{\frac{\partial}{\partial\theta^\alpha}, \frac{\partial}{\partial\bar\theta^{\dot\beta}}\right\} = 0$

Also recall $\theta Q \equiv \theta^\alpha Q_\alpha$, $\bar\theta\bar Q \equiv \bar\theta_{\dot\alpha}\bar Q^{\dot\alpha}$.

When we move θ^β across Q_α, introduce (-1) for anticommutation

[Aside: $\theta Q \equiv \theta^\alpha Q_\alpha = \epsilon^{\alpha\beta}\theta_\beta Q_\alpha = -\epsilon^{\alpha\beta}Q_\alpha\theta_\beta = +\epsilon^{\beta\alpha}Q_\alpha\theta_\beta$
$= Q^\beta\theta_\beta = Q\theta$

We can only use $\epsilon^{\alpha\beta}, \epsilon_{\alpha\beta}$ as ~~lowering~~ raising & lowering for matching second index: $\epsilon^{\alpha\beta}Q_\alpha = -\epsilon^{\beta\alpha}Q_\alpha = -Q^\beta \neq Q_\beta$]

Continue commutator:

Ref. Aitchison p.56

$$= [-\theta Q, \bar\epsilon\bar Q] + [-\bar\theta\bar Q, \epsilon Q]$$

$$= -\theta^\alpha Q_\alpha \bar\epsilon_{\dot\beta}\bar Q^{\dot\beta} + \bar\epsilon_{\dot\beta}\bar Q^{\dot\beta}\theta^\alpha Q_\alpha - \bar\theta_{\dot\beta}\bar Q^{\dot\beta}\epsilon^\alpha Q_\alpha + \epsilon^\alpha Q_\alpha \bar\theta_{\dot\beta}\bar Q^{\dot\beta}$$

$$= +\theta^\alpha\bar\epsilon_{\dot\beta} Q_\alpha \bar Q^{\dot\beta} + \theta^\alpha\bar\epsilon_{\dot\beta}\bar Q^{\dot\beta}Q_\alpha + \bar\theta_{\dot\beta}\epsilon^\alpha\bar Q^{\dot\beta}Q_\alpha + \bar\theta_{\dot\beta}\epsilon^\alpha Q_\alpha\bar Q^{\dot\beta}$$

$$= \theta^\alpha\bar\epsilon_{\dot\beta}\{Q_\alpha, \bar Q^{\dot\beta}\} + \bar\theta_{\dot\beta}\epsilon^\alpha\{\bar Q^{\dot\beta}, Q_\alpha\}$$

$$= 2\theta^\alpha\bar\epsilon_{\dot\beta} P_\mu (\sigma^\mu)_\alpha{}^{\dot\beta} + 2\bar\theta_{\dot\beta}\epsilon^\alpha P_\mu (\sigma^\mu)^{\dot\beta}{}_\alpha$$

$$= -2P_\mu\,\theta\sigma^\mu\bar\epsilon + 2P_\mu\,\epsilon\sigma^\mu\bar\theta$$

convention is: $\theta^\alpha\sigma^\mu_{\alpha\dot\beta}\bar\epsilon^{\dot\beta}$

$\epsilon_\alpha\bar\sigma^{\mu\alpha\dot\beta}\bar\theta_{\dot\beta}$

$\bar\xi\bar\sigma^\mu\chi = -\chi\sigma^\mu\bar\xi$

Then

$$\exp(A)\exp(B) = \exp[\,i(-x^\mu P_\mu - a^\mu P_\mu + i\theta\sigma^\mu\bar\epsilon P_\mu - i\epsilon\sigma^\mu\bar\theta P_\mu$$
$$+ (\theta+\epsilon)Q + (\bar\theta+\bar\epsilon)\bar Q)\,]$$

Note extra commutators vanish since $[A,B] \propto P_\mu$.

Composition of group elements gives

$G(x^\mu + a^\mu + i\epsilon\sigma^\mu\bar\theta - i\theta\sigma^\mu\bar\epsilon,\ \theta+\epsilon,\ \bar\theta+\bar\epsilon)$ as desired.

We can then see that supercoordinate translations (turn off a^μ) are generated by $i(\epsilon Q + \bar{\epsilon}\bar{Q})$.

If we represent vectors by

$$x^\mu = \frac{1}{2} x_{\alpha\dot{\alpha}} (\bar{\sigma}^\mu)^{\dot{\alpha}\alpha}$$

and $\quad x_{\alpha\dot{\alpha}} = x_\mu (\sigma^\mu)_{\alpha\dot{\alpha}}$

Recall vectors transform in $(\frac{1}{2}, \frac{1}{2})$ rep. of Lorentz

then a supertranslation by $i(\epsilon Q + \bar{\epsilon}\bar{Q})$ gives

$$\{x^\mu, \theta^\alpha, \bar{\theta}^{\dot{\alpha}}\} \to \{x^\mu + \delta x^\mu, \theta^\alpha + \delta\theta^\alpha, \bar{\theta}^{\dot{\alpha}} + \delta\bar{\theta}^{\dot{\alpha}}\}$$

with $\delta\theta^\alpha = \epsilon^\alpha$, $\delta\bar{\theta}^{\dot{\alpha}} = \bar{\epsilon}^{\dot{\alpha}}$, and

$$\delta x_{\alpha\dot{\alpha}} = -2i\theta_\alpha \bar{\epsilon}_{\dot{\alpha}} - 2i\bar{\theta}_{\dot{\alpha}}\epsilon_\alpha$$

Check:

$$x^\mu = \frac{1}{2} x_{\alpha\dot{\alpha}} (\bar{\sigma}^\mu)^{\dot{\alpha}\alpha}$$
$$= \frac{1}{2}\left(x_\mu (\sigma^\mu)_{\alpha\dot{\alpha}}\right) \bar{\sigma}^{\mu\dot{\alpha}\alpha}$$

From Shifman, (45.23), $(\sigma^\mu)_{\gamma\dot{\delta}} (\bar{\sigma}_\mu)^{\dot{\beta}\alpha} = 2\delta^{\dot{\beta}}_{\dot{\delta}}\delta^\alpha_\gamma$

$$x^\mu = x^\mu \quad \checkmark$$

Ref. Shifman, p. 423

It turns out that we can construct two invariant subspaces spanned by $\{x_L^\mu, \theta^\alpha\}$ and $\{x_R^\mu, \bar{\theta}^{\dot{\alpha}}\}$.

with
$$(x_L)_{\alpha\dot{\alpha}} = x_{\alpha\dot{\alpha}} - 2i\theta_\alpha\bar{\theta}_{\dot{\alpha}}$$
$$(x_R)_{\alpha\dot{\alpha}} = x_{\alpha\dot{\alpha}} + 2i\theta_\alpha\bar{\theta}_{\dot{\alpha}}$$

& $\delta\theta^\alpha = \epsilon^\alpha$

$$\delta(x_L)_{\alpha\dot{\alpha}} = -4i\theta_\alpha\bar{\epsilon}_{\dot{\alpha}}$$

$$\delta\bar{\theta}^{\dot{\alpha}} = \bar{\epsilon}^{\dot{\alpha}}$$
$$\delta(x_R)_{\alpha\dot{\alpha}} = -4i\bar{\theta}_{\dot{\alpha}}\epsilon_\alpha$$

Supertranslations do not reintroduce $\bar{\theta}$, θ dependence, respectively

In vector notation,

$$x_L^\mu = x^\mu - i\theta^\alpha(\sigma^\mu)_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}$$
$$x_R^\mu = x^\mu + i\theta^\alpha(\sigma^\mu)_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}$$

Will be useful when we start characterizing superfields.

Immediate consequences of SUSY.

1.) Vacuum energy vanishes. $\rightarrow$ no C.C.

Recall $\{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} = 2 P_\mu (\sigma^\mu)_{\alpha\dot{\alpha}}$

Contract & take trace on RHS: only $\sigma^0 = \mathbb{1}_{2\times 2}$ is nonzero.

$$\frac{1}{4} \sum_\alpha \left[Q_\alpha (Q_\alpha)^\dagger + (Q_\alpha)^\dagger Q_\alpha \right] = P^0$$

Sandwich between vacuum states to get vacuum energy.

$$\langle 0 | P^0 | 0 \rangle \equiv E_{vac} = \frac{1}{4} \sum_\alpha \langle 0 | Q_\alpha (Q_\alpha)^\dagger + (Q_\alpha)^\dagger Q_\alpha | 0 \rangle$$

For unbroken SUSY, Q_α and $(Q_\alpha)^\dagger$ annihilate the vacuum, $Q_\alpha |0\rangle = 0$ and $(Q_\alpha)^\dagger |0\rangle = 0$ (analogous to a regular unbroken symmetry generator acting on vacuum)

Likewise, for $Q_\alpha |0\rangle \neq 0$, $(Q_\alpha)^\dagger |0\rangle \neq 0$, then $E_{vac} > 0$ and SUSY is spontaneously broken.

$E_{vac} = 0$ is necessary and sufficient for unbroken SUSY.

2.) Bosons and fermions are mass degenerate.

Will see explicitly how $Q, \bar{Q} |\text{boson}\rangle = |\text{fermion}\rangle$ and vice versa. Since H commutes with Q, then the mass eigenvalue is a good quantum #.

3.) Equal # of bosonic + fermionic dof. in supermultiplets.

See Shifman, 47.5 + 47.6. Related, spin is not conserved as rep. of SUSY algebra; instead, particles of different spins are collected into reps. known as supermultiplets.

Comment:

A non-trivial interpretation of how SUSY evades Coleman-Mandula theorem is that it forces a relationship (namely, equality, up to a possible phase) between amplitudes for scalars and fermions. Other usual S-matrix restrictions lose analyticity, for example.

[Reference for this aside is missing.]

Back to superfields.

Superfields act on superspace $(x, \theta, \bar{\theta})$.

It is easier to understand interactions and dynamics + calculate real-world phenomenology by studying components of superfields, which are the more familiar scalar, fermion, + vector fields. Then, SUSY is manifest as symmetry operators that take certain components into others.

Most general function / superfield on $(x, \theta, \bar{\theta})$:

$$S(x,\theta,\bar{\theta}) = \phi + \theta\psi + \bar{\theta}\bar{\chi} + \theta^2 F + \bar{\theta}^2 G$$
$$+ \theta^\alpha A_{\alpha\dot{\beta}} \bar{\theta}^{\dot{\beta}} + \theta^2(\bar{\theta}\bar{\lambda}) + \bar{\theta}^2(\theta\rho)$$
$$+ \theta^2\bar{\theta}^2 D.$$

Note $A_{\alpha\dot{\beta}} = A_\mu (\sigma^\mu)_{\alpha\dot{\beta}}$

Here, $\phi, \psi, \bar{\chi}, \ldots D$ are now only dependent on x.

Note the highest power of $\theta, \bar{\theta}$ is

$\theta^2\bar{\theta}^2 \equiv \theta^\alpha\theta_\alpha \bar{\theta}_{\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}$, since any extra θs or $\bar{\theta}$s will vanish.

p.6

There are two special types of superfields that obey special properties.

First, recall the invariant subspace spanned by $\{x_L^\mu, \theta^\alpha\}$.

Transforming by $i(\epsilon Q + \bar{\epsilon}\bar{Q})$ gave $\delta\theta^\alpha = \epsilon^\alpha$,

$\delta(x_L)_{\alpha\dot{\alpha}} = -4i\theta_\alpha \bar{\epsilon}_{\dot{\alpha}}$.

To study the behavior of supertranslations on superfields,

$$\delta S = i(\epsilon Q + \bar{\epsilon}\bar{Q})S,$$

it is useful to write Q and $\bar{Q}$ as differential operators acting on superspace,

$$Q_\alpha = -\frac{i\partial}{\partial\theta^\alpha} + \bar{\theta}^{\dot{\alpha}}(\sigma^\mu)_{\alpha\dot{\alpha}}\partial_\mu$$

$$\bar{Q}_{\dot{\alpha}} = +\frac{i\partial}{\partial\bar{\theta}^{\dot{\alpha}}} - \theta^\alpha(\sigma^\mu)_{\alpha\dot{\alpha}}\partial_\mu$$

Shifman:

$\partial_{\alpha\dot{\alpha}} \equiv (\sigma^\mu)_{\alpha\dot{\alpha}}\partial_\mu$

And then $\{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} = 2P_\mu(\sigma^\mu)_{\alpha\dot{\alpha}} = 2i(\sigma^\mu)_{\alpha\dot{\alpha}}\partial_\mu$

With these differential operators, we can defines "superderivatives" which anticommute with Q_α and $\bar{Q}_{\dot{\alpha}}$:

$$D_\alpha = \frac{\partial}{\partial\theta_\alpha} - i\bar{\theta}^{\dot{\alpha}}(\sigma^\mu)_{\alpha\dot{\alpha}}\partial_\mu$$

$$\bar{D}_{\dot{\alpha}} = \frac{-\partial}{\partial\bar{\theta}^{\dot{\alpha}}} + i\theta^\alpha(\sigma^\mu)_{\alpha\dot{\alpha}}\partial_\mu$$

$\{D_\alpha, \bar{D}_{\dot{\alpha}}\} = 2i(\sigma^\mu)_{\alpha\dot{\alpha}}\partial_\mu$

Note that $\bar{D}_{\dot{\alpha}}\, x_L^\mu = 0$ and $D_\alpha\, x_R^\mu = 0$

Check: $\bar{D}_{\dot{\alpha}} x_L^\mu = \left(\frac{-\partial}{\partial\bar{\theta}^{\dot{\alpha}}} + i\theta^\alpha(\sigma^\mu)_{\alpha\dot{\alpha}}\partial_\mu\right)\left(x^\mu - i\theta^\alpha(\sigma^\mu)_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\right)$

$$= i\theta^\alpha(\sigma^\mu)_{\alpha\dot{\alpha}} + \frac{i\partial}{\partial\bar{\theta}^{\dot{\alpha}}}\left(\theta^\alpha(\sigma^\mu{}_{\alpha\dot{\alpha}})\bar{\theta}^{\dot{\alpha}}\right)$$

Have to anticommute through θ^α in second term,

$$= i\theta^\alpha(\sigma^\mu)_{\alpha\dot{\alpha}} - i\theta^\alpha(\sigma^\mu)_{\alpha\dot{\alpha}} = 0 \quad \checkmark$$

When we now shift $\partial_\mu \equiv \frac{\partial}{\partial x^\mu}$ to $\partial_\nu = \frac{\partial}{\partial x_L^\nu}$,

the superderivatives become

For $\{x_L^\mu, \theta^\alpha\}$: $\bar{D}_{\dot\alpha} = -\frac{\partial}{\partial \bar\theta^{\dot\alpha}}$, $D_\alpha = \frac{\partial}{\partial \theta^\alpha} - 2i\bar\theta^{\dot\alpha}(\sigma^\mu)_{\alpha\dot\alpha}\frac{\partial}{\partial x_L^\mu}$

In the space $\{x_R^\mu, \bar\theta^{\dot\alpha}\}$,

$$D_\alpha = +\frac{\partial}{\partial\theta^\alpha}, \quad \bar{D}_{\dot\alpha} = \frac{\partial}{\partial\bar\theta^{\dot\alpha}} + 2i\theta^\alpha(\sigma^\mu)_{\alpha\dot\alpha}\frac{\partial}{\partial x_R^\mu}$$

Note distinction between $\frac{\partial}{\partial\theta^\alpha}$ and D_α; same for $\frac{\partial}{\partial\bar\theta^{\dot\alpha}}$ and $\bar{D}_{\dot\alpha}$: The D_α, $\bar{D}_{\dot\alpha}$ acting on superfields remain superfields, while $\frac{\partial}{\partial\theta^\alpha}S$ and $\frac{\partial}{\partial\bar\theta^{\dot\alpha}}S$ are not superfields.

Martin, Section 4.3.

D_α and $\bar{D}_{\dot\alpha}$ are SUSY covariant.

Will discuss more when we consider interactions.

First type of superfield:

Chiral (LH) superfield.

Impose condition $\bar{D}_{\dot\alpha}\Phi = 0$ in $\{x_L^\mu, \theta^\alpha\}$ space.

If $\Phi(x_L, \theta)$ satisfies $\bar{D}_{\dot\alpha}\Phi = 0$, then eliminate all $\bar\theta^{\dot\alpha}$ in general superfield S. Easy to write

$$\Phi(x_L, \theta) = \phi(x_L) + \sqrt{2}\,\theta^\alpha\psi_\alpha(x_L) + \theta^2 F(x_L) \qquad (\star)$$

We identify chiral superfield is composed of a complex scalar ϕ (2 dof), Weyl fermion ψ (2 dof), and auxilliary field F (which will be non-propagating/ vanish on-shell).

When we apply supertransformations,

Shifman, eq. 48.20

$$\Phi + \delta\Phi = \phi(x_L + \delta x_L) + \sqrt{2}(\theta^\alpha + \delta\theta^\alpha)\psi_\alpha(x_L + \delta x_L)$$
$$+ (\theta^\alpha + \delta\theta^\alpha)(\theta_\alpha + \delta\theta_\alpha) F(x_L + \delta x_L),$$

and we get

$$= \phi(x_L) + [\partial_\mu \phi(x_L)]\, 2i\, \bar{\epsilon}_{\dot\alpha} (\bar\sigma^\mu)^{\dot\alpha\alpha} \theta_\alpha$$
$$+ \sqrt{2}\, \theta^\alpha \psi_\alpha(x_L) + \sqrt{2}\, \epsilon^\alpha \psi_\alpha(x_L) + \sqrt{2}\, \theta^\alpha (\partial_\mu \psi_\alpha(x_L)) \cdot 2i\, \bar\epsilon_{\dot\alpha} (\bar\sigma^\mu)^{\dot\alpha\beta}\theta_\beta$$
$$+ \theta^2 F(x_L) + 2\theta^\alpha \epsilon_\alpha F(x_L)$$

Matching powers of θ, using identity $\bar\epsilon \bar\sigma^\mu \theta = -\theta \sigma^\mu \bar\epsilon$, we get

$$\delta\phi = \sqrt{2}\, \epsilon^\alpha \psi_\alpha(x_L)$$
$$\delta\psi = -\sqrt{2}\, i (\sigma^\mu)_{\alpha\dot\alpha} (\partial_\mu \phi)\, \bar\epsilon^{\dot\alpha} + \sqrt{2}\, \epsilon_\alpha F$$
$$\delta F = i\sqrt{2}\, ((\sigma^\mu)_{\alpha\dot\alpha} \partial_\mu \psi^\alpha) \bar\epsilon^{\dot\alpha}$$

[Note: total derivative!]

And so, component scalars + fermions transform into each other under SUSY transformations!

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Vector superfield.

Impose condition of a real superfield: $V = V^\dagger$.

Get
$$V(x,\theta,\bar\theta) = C + i\theta\chi - i\bar\theta\bar\chi + \frac{i\theta^2}{\sqrt{2}} M - \frac{i\bar\theta^2}{\sqrt{2}} \bar M$$
$$- 2\theta^\alpha \bar\theta^{\dot\alpha} v_{\alpha\dot\alpha} + \left(2i\,\theta^2 \bar\theta_{\dot\alpha} \left(\bar\lambda^{\dot\alpha} - \frac{i}{4} \partial^{\dot\alpha\alpha} \chi_\alpha\right) + h.c.\right)$$
$$+ \theta^2 \bar\theta^2 \left(D - \frac{1}{4}\partial^2 C\right)$$

$$\partial^{\dot\alpha\alpha} = (\bar\sigma^\mu)^{\dot\alpha\alpha} \partial_\mu$$
$$v^\mu = \frac{1}{2} (\bar\sigma^\mu)^{\dot\alpha\alpha} v_{\alpha\dot\alpha}$$

Bosonic fields C, D, v^μ are real, others complex + bar denotes complex conjugation.

Will be able to eliminate many components $C, \chi, \bar{\chi}, M, \bar{M}$ by choosing Wess-Zumino gauge (still allows non-SUSY gauge transformations).

$$\Rightarrow \quad V(x,\theta,\bar\theta) = -2\theta^\alpha \bar\theta^{\dot\alpha} A_\mu (\sigma^\mu)_{\alpha\dot\alpha}$$
$$-2i\bar\theta^2(\theta\lambda) + 2i\theta^2(\bar\theta\bar\lambda)$$
$$+\theta^2\bar\theta^2 D.$$

Exercise: Get SUSY transformations for V.

Shifman, Problem 14.3.

Note: Errata lists typos in textbook (fixed here)

$$\delta C = i(\epsilon\chi - \bar\epsilon\bar\chi)$$
$$\delta\chi_\alpha = \sqrt{2}\, M\epsilon_\alpha + 2i v_\mu (\sigma^\mu)_{\alpha\dot\alpha}\bar\epsilon^{\dot\alpha} - (\partial_{\alpha\dot\alpha} C)\bar\epsilon^{\dot\alpha}$$
$$\delta M = 2\sqrt{2}\,\bar\epsilon_{\dot\alpha}\bar\lambda^{\dot\alpha} - i\sqrt{2}\,\bar\epsilon_{\dot\alpha}\partial^{\alpha\dot\alpha}\chi_\alpha$$
$$\delta v_{\alpha\dot\alpha} = \left[-\tfrac{1}{2}\epsilon^\beta(\partial_{\beta\dot\alpha}\chi_\alpha) + \tfrac{1}{2}\epsilon_\alpha\partial_{\beta\dot\alpha}\chi^\beta - 2i\epsilon_\alpha\bar\lambda_{\dot\alpha}\right] + \text{h.c.}$$
$$\delta\lambda_\alpha = i\epsilon_\alpha D + \tfrac{1}{2}\epsilon_\beta\partial^{\dot\alpha\beta}v_{\alpha\dot\alpha} - \tfrac{1}{2}\epsilon^\beta\partial_{\alpha\dot\beta}v^{\dot\beta}{}_\beta$$
$$\delta D = \epsilon^\alpha\partial_{\alpha\dot\alpha}\bar\lambda^{\dot\alpha} + \text{h.c.}$$

$\hookrightarrow$ again, total derivative.