Lecture 13.

SUSY

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(Thursday)

Continue SQCD.

+ a-theorem & c-theorem

Another check of Seiberg's conjectured dual model.

Add a mass term for one flavor.

$$W = m \bar{\Phi}^F \Phi_F$$

In the dual theory, we have

$$W_d = \lambda \tilde{M}_i^{\ j} \bar{\phi}^i \phi_j + c m \tilde{M}_F^F$$

using the mapping

$$M \leftrightarrow \tilde{M}$$

$$B_{i_1 \ldots i_N} \leftrightarrow \epsilon_{i_1 \ldots i_N, j_1 \ldots j_{F-N}} b^{j_1 \ldots j_{F-N}}$$

$$\bar{B}^{i_1 \ldots i_N} \leftrightarrow \epsilon^{i_1 \ldots i_N j_1 \ldots j_{F-N}} \bar{b}_{j_1 \ldots j_{F-N}}$$

$$\left(\begin{aligned} \text{Originally,} \quad M_i^{\ j} &= \bar{\Phi}^{jn} \Phi_{ni} \\ B_{i_1 \ldots i_N} &= \Phi_{n_1 i_1} \ldots \Phi_{n_N i_N} \epsilon^{n_1 \ldots n_N} \\ \bar{B}^{i_1 \ldots i_N} &= \bar{\Phi}^{n_1 i_1} \ldots \bar{\Phi}^{n_N i_N} \epsilon_{n_1 \ldots n_N} \end{aligned} \right)$$

Can identify

$\lambda \tilde{M} = \frac{M}{\mu}$, since M has same quantum numbers in both theories, which trades λ for scale μ.

The dual superpotential is

$$W_d = \frac{1}{\mu} M_i^{\ j} \bar{\phi}^i \phi_j + m M_F^{\ F}$$

Solve EOM for M_F^F:

$$\frac{\partial W_d}{\partial M_F^F} = \frac{1}{\mu} \bar{\phi}^F \phi_F + m = 0$$

$$\Rightarrow \bar{\phi}^F \phi_F = -\mu m$$

These dual squarks thus develop a vev.

This theory then has one less color, one less flavor + some singlets. It turns out (Terning, Sect. 10.5) that there is then a consistent mapping between the two dual theories when one flavor of original squarks + antisquarks gets D-flat vevs, + the corresponding meson vev gives a mass to the dual quarks + squarks.

Consider SQCD with $F=N$: Confinement with chiral sym. breaking

Seiberg: All 't Hooft anomaly matching conditions can be solved with just color singlet meson and baryons: no dual quarks. Theory is confining + all massless dof are color singlets.

Parametrize moduli space by mesons + baryons.

Baryons are now flavor singlets:

$$B = \epsilon^{i_1 \dots i_F} B_{i_1 \dots i_F}$$

$$\bar{B} = \epsilon_{j_1 \dots j_F} \bar{B}^{j_1 \dots j_F}$$

Classically, we have a constraint:

$$\det M = B\bar{B}$$

Recall, with quark masses turned on, the EOM of $M_i{}^j$ was

$$\langle M_i{}^j \rangle = (m^{-1})_i{}^j \left(\det m \, \Lambda^{3N-F}\right)^{1/N}$$

Take det of both sides, set $F=N$

$\Rightarrow$ $\det\langle M\rangle = \det(m^{-1})\det(m)/(\Lambda^{2N})$

$= \Lambda^{2N}$, which is independent of m.

However, nonzero m sets $\langle B\rangle = \langle\bar{B}\rangle = 0$ because we can integrate out all fields with baryon number $\Rightarrow$ classical constraint broken at quantum level.

Using holomorphy + symmetries,

	$U(1)_A$	$U(1)$	$U(1)_R$
$\det M$	$2N$	0	0
B	N	N	0
$\bar{B}$	N	$-N$	0
Λ^{2N}	$2N$	0	0

we construct the new constraint

$$\det M - \bar{B}B = \Lambda^{2N}\left(1 + \sum_{pq} C_{pq}\frac{(\Lambda^{2N})^p(\bar{B}B)^q}{(\det M)^{p+q}}\right)$$

where $p,q>0$ in order to ensure proper behavior for $\Lambda\to 0$ & $B,\bar{B}\to 0$.

For $\langle\bar{B}B\rangle \gg \Lambda^{2N}$, the theory is pert., but then we would have $\det M \sim (\bar{B}B)^{(q-1)/p+q}$ which is wrong in the $\Lambda\to 0$ weak coupling limit, so all $C_{pq}=0$. We get $\det M - \bar{B}B = \Lambda^{2N}$.

Note $e^{-S_{inst}} \propto \Lambda^b = \Lambda^{2N}$ has the right scaling of an instanton effect.

This theory exhibits complementarity between Higgsed & confining phases.

(A) $M_i{}^j = \Lambda^2 \delta_i{}^j$, $B = \bar{B} = 0$

Global $SU(F) \times SU(F) \times U(1) \times U(1)_R$ broken to $SU(F)_d \times U(1) \times U(1)_R$.

(B) $B\bar{B} = -\Lambda^{2N}$, $M = 0$

Global sym. breaks to $SU(F) \times SU(F) \times U(1)_R$, baryon # spont. broken. Result of squark vevs for massive squarks & gauginos.

No points where gluons are light, moduli space is smooth. No phase transition between Higgsed & confining phases.

Can also study $F = N+1$.

Terning, 10.9. s-confining theory: Exhibits confinement but does not spont. break global syms., develops dynamical superpotential.

$$W = \frac{1}{\Lambda^{2N-1}} \left(B^i M_i{}^j \bar{B}_j - \det M \right)$$

Conformal field theory.

First experience with CFT.

Generators:

$M_{\mu\nu} = -i(x_\mu \partial_\nu - x_\nu \partial_\mu)$, $P_\mu = -i\partial_\mu$

$K_\mu = -i(x^2 \partial_\mu - 2x_\mu x_\alpha \partial^\alpha)$, $D = i x_\alpha \partial^\alpha$

$\hookrightarrow$ special conformal $\qquad$ $\llcorner$ Dilation

Isomorphic to $SO(4,2)$

Given the commutator

$$i[D, P^\alpha] = P^\alpha,$$

we have $e^{i\alpha D} P^2 e^{-i\alpha D} = e^{2\alpha} P^2$,

so all masses vanish or mass spectrum is continuous. Then, S-matrix of on-shell scattering amplitudes is subsumed by n-pt. correlation functions of type

$\langle O_1(x_1) \ldots O_n(x_n) \rangle$, whose dependence on $x_i - x_j$ is power-like. These powers are critical exponents.

Scale invariance vs. conformal invariance.

$$S^\mu = x_\nu T^{\mu\nu}$$

$$C^\mu = [b_\nu x^2 - 2x_\nu (bx)] T^{\mu\nu}$$

for $x \to x'^\mu = x^\mu + \epsilon^\mu(x)$,

$\epsilon^\mu(x) = b^\mu x^2 - 2x^\mu (bx)$, b^μ const. vector.

Can show $\dfrac{x'^\mu}{x'^2} = \dfrac{x^\mu}{x^2} + b^\mu$.

$T^{\mu\nu}$ can be "improved": for example,

$$T^{\mu\nu} \mathrel{+}= \text{const.} \cdot (g^{\mu\nu} \partial^2 - \partial^\mu \partial^\nu) \phi^\dagger \phi .$$

If traceless tensor exists, $\partial_\mu S^\mu = 0 \Rightarrow T^\mu_\mu = 0$ + $\partial_\mu C^\mu = 0$. Absence of traceless energy-momentum tensor is equivalent to absence of conformal sym.

RG-flow:

c-theorem + a-theorem.

Zamolodchikov (1986) proved existence of a fcn. in 2D QFT that is monotonically decreasing under RG transformation. Fcn. is const. only at fixed points, and counts # of massless d.o.f. at that length scale.

Intuitively, RG flow always leads to loss of information about short-distance dofs.

Follow Hardy (1988):

c-fcn. given by conformal anomaly number

2D: $\langle T(z)\, T(0)\rangle = \frac{c}{2} \cdot z^{-4}$

4D: $\langle T_{\mu\nu}(r)\, T_{\lambda\sigma}(0)\rangle = \frac{A}{r^{2d+4}} r_\mu r_\nu r_\lambda r_\sigma$

$+ \frac{B}{r^{2d+2}}(r_\mu r_\nu \delta_{\lambda\sigma} + r_\lambda r_\sigma \delta_{\mu\nu})$

$+ \frac{C}{r^{2d+2}}(r_\mu r_\lambda \delta_{\nu\sigma} + r_\nu r_\lambda \delta_{\mu\sigma} + r_\mu r_\sigma \delta_{\nu\lambda} + r_\nu r_\sigma \delta_{\mu\lambda})$

$+ \frac{D}{r^{2d}} \delta_{\mu\nu}\delta_{\lambda\sigma} + \frac{E}{r^{2d}}(\delta_{\mu\lambda}\delta_{\nu\sigma} + \delta_{\nu\lambda}\delta_{\mu\sigma})$

Can construct $\tilde{c} = \frac{-4}{d-1}\left(A + \frac{1}{2}(d^2+d+2)B + (d+3)C + \frac{1}{2}d(d+1)D + (d+1)E\right)$

$+ \quad \dot{\tilde{c}} = -\frac{4d+1}{d-1} \langle \Theta\, \Theta\rangle - 2(d-2)B$

For conformally invariant point, Θ vanishing $\Rightarrow \tilde{c} = A > 0$.

Pf: Komargodski + Schwimmer: 2011, by studying 4-pt. graviton amplitudes.

Summary:

SUSY is a powerful tool to study & understand more general properties of field theory.

Two sides were emphasized in this course: pheno / applied, formal / model-building / mathematical. We have seen that some open questions in regular 4D QFT are solved in SUSY (like Yang-Mills theory & QCD), where non-pert. calculations are under control. We even have ansätze for dualities that satisfy non-trivial consistency conditions.

We also touched on CFTs, which can be considered as progenitor field theories. In the future, we can hope that some open problems in field theory or S-matrix amplitudes can be solved by intuition from how these questions are understood & tackled in SUSY.

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