SUSY

Lecture 12

Felix Yu

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't Hooft Anomaly Matching, Phases of Gauge Theory, Seiberg Duality.

First, revisit ADS superpotential.

$SU(N)$ gauge theory with

$\Phi \sim \square$ + $\bar{\Phi} \sim \bar{\square}$, $W_{tree} = 0$, calculated non-perturbative superpotential correction

$$W_{ADS} = C_{N,F} \left(\frac{\Lambda^{3N-F}}{\det M} \right)^{\frac{1}{N-F}}$$

with $M_i^j = \bar{\Phi}^{jn} \Phi_{ni}$ + $\langle \bar{\Phi}^* \rangle = \langle \Phi \rangle = \begin{pmatrix} v_1 & & \\ & \ddots & \\ & & v_F \\ 0 & \cdots & 0 \\ \vdots & & \vdots \\ 0 & \cdots & 0 \end{pmatrix}$ on the moduli space (SUSY preserving, breaks $SU(N)$ to $SU(N-F)$.)

Consider adding mass m to one flavor. This deforms the theory to $SU(N)$ with $F-1$ light (massless) flavors.

Below m, the β-fcn. changes from $3N-F$ to $3N-F+1$, which imposes the matching condition $\left(\frac{\Lambda}{m}\right)^b = \left(\frac{\Lambda_L}{m}\right)^{b_L}$ for $b = 3N-F$, $b_L = 3N-F+1$.

We get $m \Lambda^{3N-F} = \left(\Lambda^{3N-F+1}\right)_{N,F-1}$.

By holomorphy, the superpotential must be of the form

$$W_{exact} = \left(\frac{\Lambda^{3N-F}}{\det M} \right)^{\frac{1}{N-F}} f(t)$$

$$t \equiv m M_F^F \left(\frac{\Lambda^{3N-F}}{\det M} \right)^{\frac{-1}{N-F}}$$

$m M^F_F$ (F-F entry in meson field) has $U(1)_A$ charge 0 & R-charge 2, t has R-charge 0.

To recover weak coupling, small mass limit, $\Lambda \to 0$, $m \to 0$, have to reproduce earlier results with added small mass term,

$f(t) = C_{N,F} + t$.

$$\Rightarrow W_{exact} = C_{N,F} \left(\frac{\Lambda^{3N-F}}{\det M} \right)^{\frac{1}{N-F}} + m M_F{}^F.$$

EOM for $M_F{}^F$ & $M^j{}_F$,

$$\frac{\partial W}{\partial M_F{}^F} = C_{N,F} \left(\frac{\Lambda^{3N-F}}{\det M} \right)^{\frac{1}{N-F}} \left(\frac{-1}{N-F} \right) \frac{\mathrm{cof}(M_F{}^F)}{\det M} + m = 0$$

$$\frac{\partial W}{\partial M^j_F} = C_{N,F} \left(\frac{\Lambda^{3N-F}}{\det M} \right)^{1/N-F} \left(\frac{-1}{N-F} \right) \frac{\mathrm{cof}(M^j{}_F)}{\det M} = 0$$

$$\mathrm{cof}(M_{ij}) = \det M'_{ij} = \det \begin{array}{cc} & i \\ \left(\begin{array}{c|c} \square & \square \\ \hline \square & \square \end{array} \right) & j \end{array}$$

These EOMs $\frac{C_{N,F}}{N-F} \left(\frac{\Lambda^{3N-F}}{\det M} \right)^{\frac{1}{N-F}} = m M^F{}_F$ Use $\det M = \mathrm{cof}(M^F_F) \cdot M^F_F$

& $\mathrm{cof}(M_i{}^F) = 0$.

Thus M has block diagonal form

$$M = \begin{pmatrix} \hat{M} & 0 \\ 0 & M^F_F \end{pmatrix}, \quad \tilde{M} \text{ is } (F-1) \times (F-1).$$

We get $W_{exact}(N, F-1) = (N-F+1) \left(\frac{C_{N,F}}{N-F} \right)^{N-F/N-F+1} \cdot \left(\frac{\Lambda_{N,F-1}^{3N-F+1}}{\det \tilde{M}} \right)^{\frac{1}{N-F+1}}$

This is the same as $W_{ADS}^{(N,F-1)}$ up to an overall constant.

For consistency, we have recursion relation

$$C_{N,F-1} = (N-F+1) \left(\frac{C_{N,F}}{N-F} \right)^{N-F/N-F+1}.$$

Given $C_{N,N-1} = 1$ (by instanton calc. for $F = N-1$),

$C_{N,F} = N-F$.

Adding masses for all flavors,

$$W_{exact} = C_{N,F}\left(\frac{\Lambda^{3N-F}}{\det M}\right)^{\frac{1}{N-F}} + m^i_j M_i^j .$$

EOM for M: $\quad M_i^j = (m^{-1})_i^j \left(\frac{\Lambda^{3N-F}}{\det M}\right)^{\frac{1}{N-F}}$

We get $\bar{\Phi}^j \Phi_i = M_i^j = (m^{-1})_i^j \left((\det m)\Lambda^{3N-F}\right)^{1/N}$

Matching holomorphic gauge coupling at thresholds,

$$\Lambda^{3N-F} \det m = \Lambda^{3N}_{N,0},$$

so $W_{eff} = N\Lambda^3_{N,0}$, and agrees w/ gaugino condensation result, setting $a=N$ (up to a phase in the $1/N$ root above).

Key: Starting with $F=N-1$, we can consistently deform by adding masses + derive the correct ADS effective superpotential for $0 \leq F < N-1$, + gaugino condensation for $F=0$.

Terning, Chap. 10

Now, what about $N=F$ + $F>N$?

Possible phases of gauge theories: Potential between two static test charges.

Coulomb: $V(r) \sim \frac{1}{r}$ (normal QED)

Free electric: $V(r) \sim \frac{1}{r\ln(r\Lambda)}$ (massless QED)

Free magnetic: $V(r) \sim \frac{\ln(r\Lambda)}{r}$ Electric + mag. charges inversely related, EM duality swaps electrons + monopoles.

Higgs: $V(r) \sim$ constant

Confining: $V(r) \sim \sigma r$. Area law confinement

For $F>N$, $SU(N)$ theory is generally completely broken on moduli space.

Again, charges & fields:

	$SU(N)$	$SU(F)$	$SU(F)$	$U(1)$	$U(1)_R$
Φ	$\square$	$\square$	1	1	$\frac{F-N}{F}$
$\bar{\Phi}$	$\bar{\square}$	1	$\bar{\square}$	-1	$\frac{F-N}{F}$

Moduli space:

$$\langle \Phi \rangle = \overbrace{\begin{pmatrix} v_1 & & & 0 & \cdots & 0 \\ & \ddots & & \vdots & & \vdots \\ & & v_N & 0 & \cdots & 0 \end{pmatrix}}^{F} \Big\} N$$

$$\langle \bar{\Phi} \rangle = \begin{pmatrix} \bar{v}_1 & & \\ & \ddots & \\ & & \bar{v}_N \\ 0 & \cdots & 0 \\ \vdots & & \vdots \\ 0 & \cdots & 0 \end{pmatrix}$$

from $V = \frac{1}{2} D^a D^a$

$$D^a = -g\left(\phi^{*in} (T^a)_n^{\ m} \phi_{mi} - \bar{\phi}^{in} (T^a)_n^{\ m} \bar{\phi}^*_{mi} \right)$$

$$\left.\begin{aligned} d^n_{\ m} &\equiv \langle \phi^{*in} \phi_{mi} \rangle \\ \bar{d}^n_{\ m} &\equiv \langle \bar{\phi}^{in} \bar{\phi}^*_{mi} \rangle \end{aligned}\right\}$$ $N \times N$ positive semi-definite Hermitian matrices of max rank F, for $F > N$ has rank N.

$$D^a = -g T^{a\ m}_{\ n} \left(d^n_{\ m} - \bar{d}^n_{\ m} \right)$$

T^a is complete basis for traceless matrices

$$D^a = 0 \Rightarrow d^n_{\ m} - \bar{d}^n_{\ m} = \rho \mathbb{I}$$

Matrix $d^n_{\ m}$ can be diag. by $SU(N)$ gauge trans.

$$\Rightarrow d = \begin{pmatrix} |v_1|^2 & & & \\ & |v_2|^2 & & \\ & & \ddots & \\ & & & |v_N|^2 \end{pmatrix}, \quad |v_i|^2 = |\bar{v}_i|^2 + \rho$$

& $\bar{d}$ also diag.

With $SU(N)$ completely broken at generic pt. in moduli space, we end up with

$$\underset{\substack{\uparrow \\ \text{original \# of massless chiral sfields}}}{2NF} - \underset{\substack{\uparrow \\ \text{\# of chiral sfields eaten by super-Higgs}}}{(N^2-1)}$$ massless chiral sfields.

Parametrize dofs as meson + baryons:

$$M_i^j = \bar{\Phi}^{j\alpha} \Phi_{\alpha i} \qquad F^2 \text{ components}$$

$$B_{i_1 \cdots i_N} = \Phi_{\alpha_1 i_1} \cdots \Phi_{\alpha_N i_N} \epsilon^{\alpha_1 \cdots \alpha_N} : \binom{F}{N} \text{ components}$$

$$\bar{B}^{i_1 \cdots i_N} = \bar{\Phi}^{\alpha_1 i_1} \cdots \bar{\Phi}^{\alpha_N i_N} \epsilon_{\alpha_1 \cdots \alpha_N} : \binom{F}{N} \text{ components}$$

Fermion partners are same with products of scalars + one fermion.

Classical relationship $\quad B_{i_1 \cdots i_N} \bar{B}^{j_1 \cdots j_N} = M^{j_1}_{[i_1} \cdots M^{j_N}_{i_N]}$ → antisymmetrization

Up to flavor trans.,

$$\langle M \rangle = \begin{pmatrix} v_1 \bar{v}_1 & & & & & \\ & \ddots & & & & \\ & & v_N \bar{v}_N & & & \\ & & & 0 & & \\ & & & & \ddots & \\ & & & & & 0 \end{pmatrix}$$

$$\langle B_{1 \cdots N} \rangle = v_1 \cdots v_N$$

$$\langle \bar{B}^{1 \cdots N} \rangle = \bar{v}_1 \cdots \bar{v}_N$$

Recall EOM for $M_i{}^j = (m^{-1})_i^j \left((\det m) \Lambda^{3N-F} \right)^{\frac{1}{N}}$

Giving large masses m_H to flavors N to F & matching gauge coupling at the mass thresholds

$$\Lambda^{3N-F} \det m_H = \Lambda^{2N+1}_{N, N-1}$$

Low energy theory has $N-1$ flavors & ADS superpotential. For m_L masses of light flavors,

$$M_i{}^j = (m_L^{-1})_i^j \left(\det m_L \, \Lambda^{2N+1}_{N, N-1} \right)^{1/N}$$

$$= (m_L^{-1})_i^j \left(\det m_L \det m_H \, \Lambda^{3N-F} \right)^{1/N}$$

Treating m_L, m_H as holomorphic params,
can keep $m^i_j \to 0$ with comps. of M finite or zero.

At large vevs, quantum moduli space spanned by $M, B, \bar{B}$ is weakly coupled & gauge theory broken perturbatively.
At small vevs, gauge couplings grow. Will find dual description by Seiberg.

Aside:

Banks-Zaks IR fixed point.
For $F \geq 3N$, lose asymp. freedom.
For $F < 3N$, just below, have IR fixed point.

Consider large N & F/N infinitesimally below pt. where asymp. freedom is lost.

Exact NSVZ β fcn.

Canonical gauge coupling
$$\beta(g) = \frac{-g^3}{16\pi^2} \frac{(3N - F(1-\gamma))}{1 - Ng^2/8\pi^2}$$

anom. dim. of quark mass
$$\gamma = -\frac{g^2}{8\pi^2} \frac{N^2-1}{N} + O(g^4)$$

$$\Rightarrow \quad 16\pi^2 \beta(g) = -g^3(3N-F) - \frac{g^5}{8\pi^2}\left(3N^2 - 2FN + \frac{F}{N}\right) + O(g^7)$$

Choose $F = 3N - \epsilon N$
$$16\pi^2\beta(g) = -g^3 \epsilon N + \frac{g^5}{8\pi^2}\left(3(N^2-1) + O(\epsilon)\right) + O(g^7)$$

Generate IR fixed pt. $\Rightarrow \quad g_*^2 = \frac{8\pi^2}{3} \frac{N}{N^2-1} \epsilon$

Gauge theory is scale invariant for $g = g_*$.
Scale invariant w/ fields w/ spin $\leq 1 \Rightarrow$ Conformal inv.

Global syms. still preserved by conformal theory.
Can apply 't Hooft anomaly matching.
Dual theory by Seiberg:

	$SU(F-N)$	$SU(F)$	$SU(F)$	$U(1)$	$U(1)_R$
q	$\square$	$\bar{\square}$	1	$N/F-N$	N/F
$\bar{q}$	$\bar{\square}$	1	$\square$	$-N/F-N$	N/F
mesino	1	$\square$	$\bar{\square}$	0	$2\frac{F-N}{F}$

dual quarks = magnetic quarks

Anomaly matching

	anomaly	dual anomaly
$SU(F)^3$	$-(F-N)+F$	N
$U(1)\,SU(F)^2$	$\frac{N}{F-N}(F-N)\frac{1}{2}$	$\frac{N}{2}$
$U(1)_R\,SU(F)^2$	$\frac{N-F}{F}(F-N)\frac{1}{2}+\frac{F-2N}{F}F\frac{1}{2}$	$\frac{-N^2}{2F}$
$U(1)^3$	0	0
$U(1)$	0	0
$U(1)\,U(1)_R^2$	0	0
$U(1)_R$	$\left(\frac{N-F}{F}\right)2(F-N)F+\frac{(F-2N)}{F}F^2+(F-N)^2-1$	$-N^2-1$
$U(1)_R^3$	$\left(\frac{N-F}{F}\right)^3 2(F-N)F+\left(\frac{F-2N}{F}\right)^3F^2+(F-N)^2-1$	$-\frac{2N^4}{F^2}+N^2-1$
$U(1)^2\,U(1)_R$	$\left(\frac{N}{F-N}\right)^2\frac{N-F}{F}2F(F-N)$	$-2N^2$

Can write unique superpotential

$$W = \lambda \underset{\substack{\uparrow \\ \text{Mesino}}}{\tilde{M}_i{}^j}\, \tilde{q}_j\, \bar{\tilde{q}}^i$$

Baryon operators:

$$b^{i_1 \dots i_{F-N}} = \tilde{q}^{n_1 i_1} \cdots \tilde{q}^{n_{F-N}\, i_{F-N}}\, \epsilon_{n_1 \dots n_{F-N}}$$

$$\bar{b}_{i_1 \dots i_{F-N}} = \bar{\tilde{q}}_{n_1 i_1} \cdots \bar{\tilde{q}}_{n_{F-N},\, i_{F-N}}\, \epsilon^{n_1 \dots n_{F-N}}$$

Mapping between theories:

$$M \leftrightarrow \tilde{M}$$

$$B_{i_1 \dots i_N} \leftrightarrow \epsilon_{i_1 \dots i_N,\, j_1 \dots j_{F-N}}\, b^{j_1 \dots j_{F-N}}$$

$$\bar{B}^{i_1 \dots i_N} \leftrightarrow \epsilon^{i_1 \dots i_N\, j_1 \dots j_{F-N}}\, \bar{b}_{j_1 \dots j_{F-N}}$$

Note $\beta(\tilde{g}) \sim -\tilde{g}^3 (3\tilde{N} - F) = -\tilde{g}^3 (2F - 3N)$

So dual theory loses asymptotic freedom for $F \leq \frac{3N}{2}$.

By superconformal arguments, can also find that the IR fixed pt. of original theory is interacting conformal theory for $\frac{3}{2} N < F < 3N$.

Original: Weakly coupled near $F = 3N$, moves to stronger coupling as F is reduced.

Dual: Interacting fixed point in same region. Weak coupling near $F = \frac{3N}{2}$ + stronger as F increased. For $F \leq \frac{3N}{2}$, IR fixed point is trivial (asymptotic freedom lost, $\tilde{g}_*^2 = 0$)

~~Key: Two theories w/~~ Can even lower to $F = N+2$.

Key: for $3N > F > \frac{3N}{2}$, have two theories with IR fixed points that describe same physics.
For $\frac{3N}{2} \geq F > N+1$, strongly coupled theory + IR free theory describe same physics. Dual theory is free, massless composite gauge bosons, quarks, mesons, + superpartners.