SUSY

Lecture 11

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Last time: Holomorphic gauge coupling.

Gaugino condensation in SYM theory.

Today: Super-QCD, ~~'t Hooft anomaly matching~~

SU(N) QCD with F flavors (there are 2NF chiral superfields), and start with $F < N$.

Theory has fields and symmetries:

	$SU(N)$	$SU(F)$	$SU(F)$	$U(1)$	$U(1)_R$
Φ	$\square$	$\square$	1	1	$\frac{F-N}{F}$
$\overline{\Phi}$	$\overline{\square}$	1	$\overline{\square}$	-1	$\frac{F-N}{F}$

$U(1)_R$ anomaly free

$1 \cdot T(Ad) + (R-1)\, T(\square)\, 2F = 0$

ansatz

$SU(F) \times SU(F)$ analogous to $SU(3)_L \times SU(3)_R$ in QCD w/ u, d, s quarks. $U(1)$ is analog of baryon #.

Quarks denoted Q + $\overline{Q}$ for Φ, $\overline{\Phi}$, respectively.

Recall R-symmetry + U(1) global = new R-symmetry.

Extra $U(1)_A$ is anomalous.

We have D-terms given by

$$D^a = g\left(\Phi^{*jn} (T^a)^m{}_n \Phi_{mj} - \overline{\Phi}^{jn} (T^a)^m{}_n \overline{\Phi}^*_{mj}\right)$$

Generically,

$$\begin{aligned}
\mathcal{L} = & -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + i \lambda^{\dagger a} \bar{\sigma}^\mu D_\mu \lambda^a + \frac{1}{2} D^a D^a \\
& + D^\mu \phi^{*j} D_\mu \phi_j + i \psi^{\dagger j} \bar{\sigma}^\mu D_\mu \psi_j \\
& - \frac{1}{2} M^{jk} \psi_j \psi_k - \frac{1}{2} M^*_{jk} \psi^{\dagger j} \psi^{\dagger k} - V(\phi, \phi^*) \\
& - \frac{1}{2} y^{jkn} \phi_j \psi_k \psi_n - \frac{1}{2} y^*_{jkn} \phi^{*j} \psi^{\dagger}_j \psi^{\dagger}_n \\
& - \sqrt{2} g \left[(\phi^* T^a \psi) \lambda^a + \lambda^{\dagger a} (\psi^\dagger T^a \phi) \right] + g (\phi^* T^a \phi) D^a
\end{aligned}$$

Recall D-term potential is

$$V = \frac{1}{2} D^a D^a .$$

We have $W=0$ (no superpotential) at the moment.

Classically, we have a moduli space

$$\langle \bar{\Phi}^* \rangle = \langle \Phi \rangle = \begin{array}{c} \overbrace{\qquad\qquad}^{F \text{ columns.}} \\ \left(\begin{array}{ccc} v_1 & & \\ & \ddots & \\ & & v_F \\ 0 & \cdots & 0 \\ \vdots & & \vdots \\ 0 & \cdots & 0 \end{array} \right) \end{array} \left. \vphantom{\begin{array}{c} a \\ a \\ a \\ a \\ a \\ a \end{array}} \right\} N \text{ rows}$$

Use gauge + global syms. to rotate to simple form.

It shows $D^a \big|_{\langle \bar{\Phi}^* \rangle = \langle \Phi \rangle} = 0$.

At a generic point in moduli space, $SU(N)$ is super-Higgsed to $SU(N-F)$.

We have $N^2 - 1 - \left((N-F)^2 - 1 \right) = 2NF - F^2$ broken generators, and so from $2NF$ chiral supermultiplets, with $2NF - F^2$ eaten, we have F^2 that are massless.

These can be written in a gauge invariant way by

(F^2 mesons) $\Rightarrow$ $M_i{}^j = \bar{\Phi}^{jn} \Phi_{ni}$, summing over color index n.

Since M is chiral superfield, by holomorphy, only renormalization of M is product of WFR of Φ + $\bar{\Phi}$.

As constructed, $U(1)_R$ is non-anomalous.

$U(1)_A$ is anomalous since no gaugino contribution.

$$A_{Agg} = 1 \cdot 2F \cdot \frac{1}{2} = F .$$

We keep account of selection rule from $U(1)_A$ anomaly by spurious symmetry:

$$\left. \begin{array}{l} Q \to e^{i\alpha} Q \\ \bar{Q} \to e^{i\alpha} \bar{Q} \end{array} \right\} LH$$

$$\theta_{YM} \to \theta_{YM} + 2F\alpha .$$

The spurious symmetry leaves path integral invariant.
Under this transformation, holomorphic intrinsic scale goes as $\Lambda^b \to e^{i2F\alpha}\Lambda^b$.

We now contruct effective superpotential from holomorphy + symmetries.
Ingredients are

	$U(1)_A$	$U(1)_R$	$U(1)_B$
$W^a W^a$	0	2	0
Λ^b	$2F$	0	0
$\det M$	$2F$	$2(F-N)$	0

$\det M$ is only $SU(F)\times SU(F)$ invariant we can create from M. We need Λ^b for ~~per~~ periodicity of θ_{YM}.

$$W = \Lambda^{bn}(W^aW^a)^m(\det M)^p$$

For neutral under $U(1)_A$, $\quad n2F + p2F = 0$

$$2m + 2(F-N)p = 2$$

We get $n = -p = \frac{1-m}{N-F}$.

Since $b = 3N-F > 0$, we can only realize a weak-coupling limit $(\Lambda \to 0)$ if $n \geq 0 \Rightarrow p \leq 0$ and (since $N > F$) $m \leq 1$. Since W^aW^a contains derivative terms, locality requires $m \geq 0$ and m is integer.
Get two terms: $m=1$ is tree-level field strength. Coefficient restricted to be prop. to $b\ln\Lambda$ by periodicity of θ_{YM}. $\Rightarrow$ gauge coupling gets no non pert. renormalizations.

Second term, $m=0$, is Affleck-Dine-Seiberg superpotential:

$$W_{ADS} = C_{N,F}\left(\frac{\Lambda^{3N-F}}{\det M}\right)^{\frac{1}{N-F}}$$

Can show, if we subsequently give masses to each flavor, resulting superpotential connects smoothly to the gaugino condensation result. (See Terning, 9.2, 9.3)

The 3-2 model by ADS.

Much interest in dynamical SUSY breaking, since scale given by dimn. transmutation is much lower the UV cutoff / Planck scale. In studies of models that break SUSY, non-perturbative effects can lift flat directions at origin of moduli space + for large VEVs.

	$SU(3)$	$SU(2)$	$U(1)$	$U(1)_R$
Q	$\square$	$\square$	$1/3$	1
L	1	$\square$	-1	-3
$\bar{U}$	$\bar{\square}$	1	$-4/3$	-8
$\bar{D}$	$\bar{\square}$	1	$2/3$	4

gauge   global

Write $\bar{Q} = (\bar{U}, \bar{D})$, use Λ_3 for $SU(3)$ + Λ_2 for $SU(2)$.

For $\Lambda_3 \gg \Lambda_2$, we get

$$W_{dyn} = \frac{\Lambda_3^7}{\det(\bar{Q}Q)}$$, which has a runaway vacuum.

Adding trilinear term $W = \frac{\Lambda_3^7}{\det(\bar{Q}Q)} + \lambda Q\bar{D}L$

removes classical flat dirs. + produces stable min.

EOM of L: $\frac{\partial W}{\partial L_\alpha} = \lambda \epsilon^{\alpha\beta} Q_{m\alpha} \bar{D}^m = 0$

Tries to set det $\bar{Q}Q$ to zero:

$$\det \bar{Q}Q = \det \begin{pmatrix} \bar{U}Q_1 & \bar{U}Q_2 \\ \bar{D}Q_1 & \bar{D}Q_2 \end{pmatrix}$$
$$= \bar{U}^m Q_{m\alpha} \bar{D}^n Q_{n\beta} \epsilon^{\alpha\beta}$$

Potential cannot have zero-energy minimum since dynamical term diverges at det $\bar{Q}Q = 0$. $\Rightarrow$ SUSY is broken!

Vacuum energy estimate: All vevs $\sim \phi$, $\phi \gg \Lambda_3$ + $\lambda \ll 1$, perturbative.

$$V = \sum |F|^2$$
$$\approx \frac{\Lambda_3^{14}}{\phi^{10}} + \frac{\lambda \Lambda_3^7}{\phi^3} + \lambda^2 \phi^4$$

Minimum is near $\langle\phi\rangle \approx \frac{\Lambda_3}{\lambda^{1/7}}$ $\ni$ at weak coupling, $\lambda \to 0$, vacuum is $\langle\phi\rangle \gg \Lambda_3$. We get

$$V \approx \lambda^{10/7} \Lambda_3^4 .$$

Next time: Seiberg duality.