SUSY

Lecture 10

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(No lecture on Jan. 17)

From last time, discussed vacuum structure + moduli; ~~Abelian~~ super-Higgs mechanism.

Today, start study of super-Yang Mills and super-QCD.

The $SU(N)$ super-Yang Mills action is the supersymmetric version of $\mathcal{L} = -\frac{1}{4g^2} G^a_{\mu\nu} G^{a\mu\nu}$, $G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + f^{abc} A^b_\mu A^c_\nu$

Recall that we embedded vector fields into vector superfields V (from super QED), $V = V^\dagger$. Moreover, in Wess-Zumino gauge, we were able to eliminate components of the vector superfield via supergauge transformations, leaving normal gauge transformation freedom. The explicit form of $V(x^\mu, \theta, \bar\theta)$ is then

$$V^a = -2\theta^\alpha \bar\theta^{\dot\alpha} A^a_{\alpha\dot\alpha} - 2i\bar\theta^2(\theta\lambda^a) + 2i\theta^2(\bar\theta\bar\lambda^a) + \theta^2\bar\theta^2 D^a,$$

and $V \equiv V^a T^a$ (make V matrix-valued).

Under supergauge transformation,

$$e^V \to e^{i\bar\Lambda(x_R, \bar\theta)} e^V e^{-i\Lambda(x_L, \theta)}$$

and any matter superfields

$$Q(x_L, \theta) \to e^{i\Lambda(x_L, \theta)} Q \quad , \Lambda \equiv \Lambda^a T^a$$

$$\bar Q(x_R, \theta) \to e^{-i\Lambda(x_R, \bar\theta)} \bar Q \quad , \bar\Lambda \equiv \bar\Lambda^a T^a$$

Note that V^3 + higher powers of V vanishes, so it is easy to expand e^V.

Before, for QED, we constructed the photon field strength ~~#~~ tensor from

$$W_\alpha \equiv \frac{1}{8}\bar{D}^2 D_\alpha V = i\left(\lambda_\alpha + i\theta_\alpha D - \theta^\beta F_{\alpha\beta} - i\theta^2 \partial_{\alpha\dot{\alpha}}\bar{\lambda}^{\dot{\alpha}}\right)$$

where

D_α, $\bar{D}_{\dot{\alpha}}$ were covariant superderivatives.

Here, to account for the non-Abelian nature, we need a gauge connection

$$W_\alpha = -\frac{1}{8}\bar{D}^2 e^{-V}\left(D_\alpha e^V\right)$$

$$= i\left(\lambda_\alpha + i\theta_\alpha D - \theta^\beta G_{\alpha\beta} - i\theta^2 D_{\alpha\dot{\alpha}}\bar{\lambda}^{\dot{\alpha}}\right)$$

gauge covariant deriv.

And

$$W^2(x_L, \theta)$$

$$= -\lambda^2 - 2i(\lambda\theta)D + 2\lambda^\alpha G_{\alpha\beta}\theta^\beta + \theta^2\left(D^2 - \frac{1}{2}G^{\alpha\beta}G_{\alpha\beta}\right) + 2i\theta^2\bar{\lambda}_{\dot{\alpha}}D^{\dot{\alpha}\alpha}\lambda_\alpha$$

If we include matter in rep. R,

$$\mathcal{L} = \left(\frac{1}{4g^2}\int d^2\theta\, W^{a\alpha}W^a_\alpha + h.c.\right)$$

$$+ \sum_f \int d^2\theta\, d^2\bar{\theta}\, \bar{Q}^f e^V Q_f$$

$$+ \left(\int d^2\theta\, W(Q_f) + h.c.\right)$$

Superpotential should be all gauge invariant terms.

$$\bar{F}_f = \left.\frac{\partial W}{\partial Q_f}\right|_{Q_f = \tilde{q}_f}$$

$$D^a = -g^2 \sum_f \bar{\tilde{q}}_f T^a \tilde{q}_f$$

$$V = \frac{1}{2g^2}D^a D^a + \sum_f |F_f|^2$$

Terning 8.4

The gauge coupling can be complexified and considered as a spurion superfield.

Write $\tau \equiv \frac{\theta_{YM}}{2\pi} + \frac{4\pi i}{g^2}$, holomorphic gauge coupling

then we can write the superpotential

$$\frac{1}{16\pi i} \int d^4x \int d^2\theta \, \tau W^a_\alpha W^{a\alpha} + h.c.$$

$$= \int d^4x \left[-\frac{1}{4g^2} F^{a\mu\nu} F^a_{\mu\nu} - \frac{\theta_{YM}}{32\pi^2} F^{a\mu\nu} \tilde{F}^a_{\mu\nu} + \frac{i}{g^2} \lambda^{a\dagger} \bar{\sigma}^\mu D_\mu \lambda^a \right.$$
$$\left. + \frac{1}{2g^2} D^a D^a \right]$$

We can "control" the effect of renormalization of the gauge coupling g by understanding the constraints of holomorphy + superpotential nonrenormalization.

First, we go to a canonically normalized basis:

$$(A^a_\mu, \lambda^a_\alpha, D^a) \to g\,(A^a_\mu, \lambda^a_\alpha, D^a)$$

In $N=1$ SUSY and $SU(N)$ theory with F flavors, the RG equation of g at 1-loop is

$$\mu \frac{dg}{d\mu} = \frac{-b}{16\pi^2} g, \qquad b = 3N - F.$$

Solving for g, $\frac{1}{g^2(\mu)} = \frac{-b}{8\pi^2} \ln\left(\frac{|\Lambda|}{\mu}\right)$,

where $|\Lambda|$ arises via dimensional transmutation.

Correspondingly,

$$\tau_{1\text{-loop}} = \frac{\theta_{YM}}{2\pi} + \frac{4\pi i}{g^2(\mu)}$$
$$= \frac{1}{2\pi i} \ln\left(\left(\frac{|\Lambda|}{\mu}\right)^b e^{i\theta_{YM}} \right)$$

If we define a holomorphic intrinsic scale

$$\Lambda \equiv |\Lambda| e^{i\theta_{YM}/b} = \mu e^{2\pi i \tau / b}$$

Then $\tau_{1\text{-loop}} = \frac{b}{2\pi i} \ln\left(\frac{\Lambda}{\mu}\right)$

This is one example of a perturbation result in SUSY calculations. We can go further by studying non-perturbative effects from θ_{YM}.

Recall $\theta_{YM} F^a \tilde{F}^a$ violates CP,

$$F^{a\mu\nu} \tilde{F}^a_{\mu\nu} = 4 \epsilon^{\mu\nu\rho\sigma} \partial_\mu \mathrm{Tr}\left[A_\nu \partial_\rho A_\sigma + \frac{2}{3} A_\nu A_\rho A_\sigma \right]$$

Since $F^a \tilde{F}^a$ is a total derivative, and we have gauge field configuration with non-trivial winding numbers (instanton configurations), the integral over $\int d^4x$ gives the winding number

$$\frac{\theta_{YM}}{32\pi^2} \int d^4x \; F^{a\mu\nu} \tilde{F}^a_{\mu\nu} = n\, \theta_{YM}.$$

In the path integral,

$$\int \mathcal{D}A^a \mathcal{D}\lambda^a \mathcal{D}D^a \, e^{iS},$$

the action S depends on θ_{YM} only via $n\theta_{YM}$, with $n \in \mathbb{Z}$, and so $\theta_{YM} \to \theta_{YM} + 2\pi$ is an exact symmetry of the theory, even non-perturbatively, since it has no effect on the path integral.

Recall that the finite action of an instanton field configuration is $S_{inst.} = \frac{8\pi^2}{g^2(\mu)}$

We have the effective superpotential

$$W_{eff} = \frac{\tau(\Lambda, \mu)}{16\pi i} W^a_\alpha W^a_\alpha$$

Because $\theta_{YM} \to \theta_{YM} + 2\pi$ is a symmetry,

$\Lambda \to e^{2\pi i/b} \Lambda$ is also a symmetry.

From $\Lambda = \mu e^{2\pi i \tau / b} \to \mu e^{2\pi i \tau/b + 2\pi i/b} = \Lambda^{2\pi i/b}$

Allowing for non-perturbative effects, we can parametrize

$$\tau(\Lambda,\mu) = \frac{b}{2\pi i} \ln\left(\frac{\Lambda}{\mu}\right) + f(\Lambda,\mu)$$

where f is a holomorphic fcn. of Λ. Since $\Lambda \to 0$ corresponds to weak coupling where we must recover the perturbative result, f has a Taylor series expansion in pos. powers of Λ. Now, requiring invariance under $\Lambda \to \Lambda e^{2\pi i/b}$, f is invariant separately (perturbative term shifts θ_{YM} by 2π, as required). So we must have a Taylor series in positive powers of Λ^b.

Generic form $\Rightarrow$

$$\tau(\Lambda,\mu) = \frac{b}{2\pi i} \ln\left(\frac{\Lambda}{\mu}\right) + \sum_{n=1}^{\infty} a_n \left(\frac{\Lambda}{\mu}\right)^{bn}$$

No running beyond one-loop for holomorphic coupling.

Note: holomorphic coupling non-renormalization beyond 1-loop is true when fields are <u>not</u> canonically normalized. Rescaling fields by $A^\mu \to g A^\mu$ introduces rescaling anomaly from wavefcn. renormalization. [Terning, Sec. 8.6].

Gaugino condensation:

$SU(N)$ theory, $F=0$, only fermion is gaugino.

The $U(1)_R$ symmetry is anomalous (gauginos have R-charge 1) from triangle diagram of $U(1)_R \times SU(N)^2$.

The chiral rotation $\lambda^a \to e^{i\alpha}\lambda^a$ is equivalent to shift in $F^a_{\mu\nu}\tilde{F}^{a\mu\nu}$, $\theta_{YM} \to \theta_{YM} - 2N\alpha$, where $2N$ comes from λ^a in adjoint rep. of gauge group & has $2N$ zero modes in a one-instanton background.

The chiral rotation is only a symmetry for $\alpha = \frac{k\pi}{N}$, and so $U(1)_R$ sym. is explicity broken to discrete $\mathbb{Z}_{2N}$ subgroup. Promoting τ to a chiral superfield spurion & under transformations

$$\lambda^a \to e^{i\alpha}\lambda^a, \quad \tau \to \tau + \frac{N\alpha}{\pi},$$

path integral is invariant.

From this restriction, effective superpotential must be

$$W_{eff} = a\mu^3 e^{2\pi i\tau/N},$$

because it preserves $W_{eff} \to e^{2i\alpha} W_{eff}$ under $U(1)_R$.

We have to assume no massless particles = assume a mass gap exists. The F-term of τ (as a spurion) acts as a source for $\lambda^a\lambda^a$, which is a gaugino condensate.

$$\langle \lambda^a\lambda^a \rangle = 16\pi i \frac{\partial}{\partial\tau} W_{eff} = 16\pi i \frac{2\pi i}{N} a\mu^3 e^{2\pi i\tau/N},$$

and neglecting the non-perturbative corrections to running τ (which only give a phase), using $b = 3N$,

$$\langle \lambda^a\lambda^a \rangle = \frac{-32\pi^2}{N} a\Lambda^3$$

Gluino condensate implies that the discrete $\mathbb{Z}_{2N}$ symmetry is spontaneously broken to N degenerate but distinct vacua. $\langle \lambda^a\lambda^a \rangle \to e^{2i\alpha} \langle \lambda^a\lambda^a \rangle$, only invariant for $\alpha = \frac{k\pi}{N}$ for $k = 0$ or N.

Next time: Add quarks & see how theory deforms.