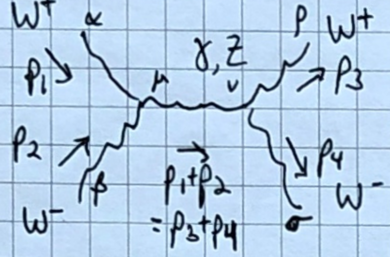
Check $W^+W^- \to W^+W^-$ matrix elements

13.11.20



$$i\mathcal{M}_{s,\gamma} = \epsilon_1^{\alpha}\,\epsilon_2^{\beta}\,\epsilon_3^{\rho *}\,\epsilon_4^{\sigma *}$$

$$(-ie)^2\left(\frac{-ig_{\mu\nu}}{s}\right)\Big((p_1-p_2)_\mu g_{\alpha\beta} + (p_2-(-p_1-p_2))_\alpha g_{\beta\mu} + (-(p_1+p_2)-p_1)_\beta g_{\mu\alpha}\Big)$$

$$\Big((-p_4+p_3)_\nu g_{\sigma\rho} + (-p_3-p_3-p_4)_\sigma g_{\rho\nu} + (p_3+p_4-(-p_4))_\rho g_{\nu\sigma}\Big)$$

$$= ie^2\frac{g_{\mu\nu}}{s}\Big((p_1-p_2)_\mu\,\epsilon_1\cdot\epsilon_2 + (2p_2-p_1)\cdot\epsilon_1\,\epsilon_{2\mu} + (-2p_1-p_2)\cdot\epsilon_2\,\epsilon_{1\mu}\Big)$$

$$\Big(-(p_3-p_4)_\nu\,\epsilon_3^*\cdot\epsilon_4^* - (2p_3+p_4)\cdot\epsilon_4^*\,\epsilon_{3\nu}^* + (p_3+2p_4)\cdot\epsilon_3^*\,\epsilon_{4\nu}^*\Big)$$

Before we use the approximate forms of ϵ, we can set $p\cdot\epsilon = 0$.

$$= -ie^2\frac{g_{\mu\nu}}{s}\Big((p_1-p_2)_\mu\,\epsilon_1\cdot\epsilon_2 + (2p_2)\cdot\epsilon_1\,\epsilon_{2\mu} - 2p_1\cdot\epsilon_2\,\epsilon_{1\mu}\Big)$$

$$\Big((p_4-p_3)_\nu\,\epsilon_3^*\cdot\epsilon_4^* + 2p_3\cdot\epsilon_4^*\,\epsilon_{3\nu} - 2p_4\cdot\epsilon_3^*\,\epsilon_{4\nu}^*\Big)$$

The Z-mediated diagram is exactly the same except for the propagator + coupling.

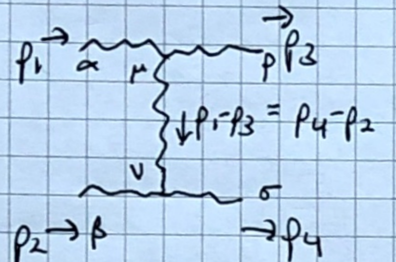
Propagator in $R_\xi = \infty$ is $\dfrac{-i}{(p_1+p_2)^2 - m_Z^2}\left(g_{\mu\nu} - \dfrac{(p_1+p_2)_\mu(p_1+p_2)_\nu}{m_Z^2}\right)$

Check: $$\frac{(p_1+p_2)_\mu(p_1+p_2)_\nu}{m_Z^2}\Big((p_1-p_2)_\mu\,\epsilon_1\cdot\epsilon_2 + 2p_2\cdot\epsilon_1\,\epsilon_{2\mu} - 2p_1\cdot\epsilon_2\,\epsilon_{1\mu}\Big) \times (\ldots)$$

$$= \Big(0 + 2p_2\cdot\epsilon_1\,\epsilon_2\cdot p_1 - 2p_1\cdot\epsilon_2\,\epsilon_1\cdot p_2\Big) = 0$$

So $$i\mathcal{M}_{s,\gamma+Z} = -ig^2\left(\frac{s_W^2}{s} + \frac{c_W^2}{s-m_Z^2}\right)\Big((p_1-p_2)_\mu\,\epsilon_1\cdot\epsilon_2 + 2p_2\cdot\epsilon_1\,\epsilon_{2\mu} - 2p_1\cdot\epsilon_2\,\epsilon_{1\mu}\Big)$$

$$\Big((p_4-p_3)^\mu\,\epsilon_3^*\cdot\epsilon_4^* + 2p_3\cdot\epsilon_4^*\,\epsilon_3^{*\mu} - 2p_4\cdot\epsilon_3^*\,\epsilon_4^{*\mu}\Big)$$



$$i\mathcal{M}_{t,\gamma} = \epsilon_1^\alpha \epsilon_2^\beta \epsilon_3^{\rho *} \epsilon_4^{\sigma *} (-ie)^2 \left(\frac{-i}{t} g_{\mu\nu}\right)$$

$$\left((-p_3-p_1)_\mu g_{\rho\alpha} + (p_1+p_1-p_3)_\rho g_{\alpha\mu} + (-p_1+p_3+p_3)_\alpha g_{\mu\rho} \right)$$

$$\left((p_2+p_4)_\nu g_{\beta\sigma} + (-p_4-p_4+p_2)_\beta g_{\sigma\nu} + (p_4-p_2-p_2)_\sigma g_{\nu\beta} \right)$$

$$= \frac{ie^2}{t} g_{\mu\nu} \left(-(p_1+p_3)_\mu \epsilon_1\cdot\epsilon_3^* + (2p_1-p_3)\cdot\epsilon_3\, \epsilon_{1\mu} + (2p_3-p_1)\cdot\epsilon_1\, \epsilon_{3\mu}^* \right)$$

$$\left((p_2+p_4)_\nu \epsilon_2\cdot\epsilon_4^* + (-2p_4+p_2)\cdot\epsilon_2\, \epsilon_{4\nu}^* + (p_4-2p_2)\cdot\epsilon_4^*\, \epsilon_{2\nu} \right)$$

Again, drop $p\cdot\epsilon = 0$

$$= \frac{ie^2}{t} g_{\mu\nu} \left(-(p_1+p_3)_\mu \epsilon_1\cdot\epsilon_3^* + 2p_1\cdot\epsilon_3\, \epsilon_{1\mu} + 2p_3\cdot\epsilon_1\, \epsilon_{3\mu}^* \right)$$

$$\left((p_2+p_4)_\nu \epsilon_2\cdot\epsilon_4^* - 2p_4\cdot\epsilon_2\, \epsilon_{4\nu}^* - 2p_2\cdot\epsilon_4^*\, \epsilon_{2\nu} \right)$$

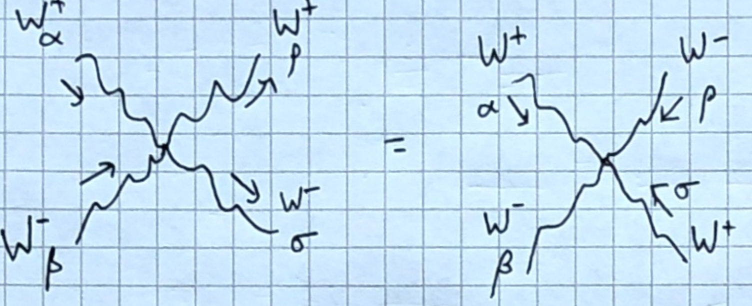
Z-mediated diagram: Check longitudinal part:

$$\frac{(p_1-p_3)_\mu (p_1-p_3)_\nu}{m_Z^2} \left(-(p_1+p_3)_\mu \epsilon_1\cdot\epsilon_3^* + 2p_1\cdot\epsilon_3\, \epsilon_{1\mu} + 2p_3\cdot\epsilon_1\, \epsilon_{3\mu}^* \right) \times (\ldots)$$

$$= \frac{(p_1-p_3)_\nu}{m_Z^2} \left(0 - 2p_1\cdot\epsilon_3\, \epsilon_1\cdot p_3 + 2p_3\cdot\epsilon_1\, p_1\cdot\epsilon_3 \right) \times (\ldots) = 0$$

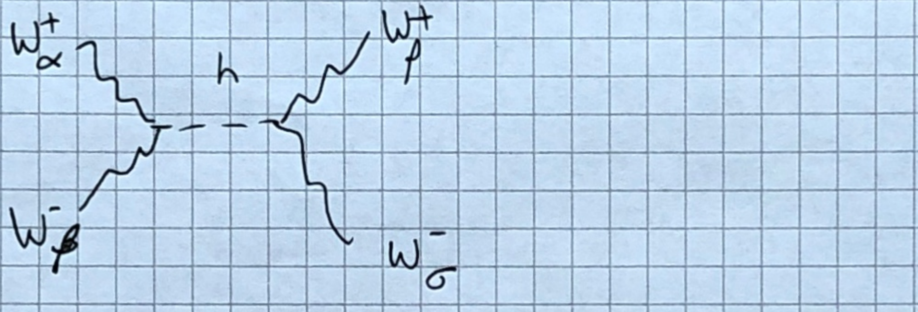
So, $$i\mathcal{M}_{t,\gamma+Z} = i g^2 \left(\frac{s_W^2}{t} + \frac{c_W^2}{t-m_Z^2} \right) \left(-(p_1+p_3)_\mu \epsilon_1\cdot\epsilon_3^* + 2p_1\cdot\epsilon_3\, \epsilon_{1\mu} + 2p_3\cdot\epsilon_1\, \epsilon_{3\mu}^* \right)$$

$$\left((p_2+p_4)^\mu \epsilon_2\cdot\epsilon_4^* - 2p_4\cdot\epsilon_2\, \epsilon_4^{*\mu} - 2p_2\cdot\epsilon_4^*\, \epsilon_2^\mu \right)$$



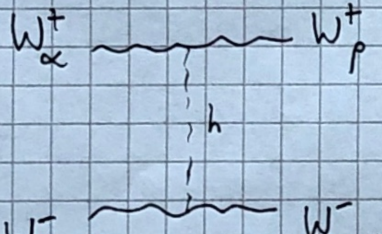
$$i\mathcal{M} = \epsilon_1^{\alpha}\epsilon_2^{\beta}\epsilon_3^{\rho *}\epsilon_4^{\sigma *}\,(ig^2)\left(2 g_{\alpha\sigma} g_{\beta\rho} - g_{\alpha\beta} g_{\rho\sigma} - g_{\alpha\rho} g_{\beta\sigma}\right)$$

$$= ig^2\left(2\epsilon_1\cdot\epsilon_4\,\epsilon_2\cdot\epsilon_3 - \epsilon_1\cdot\epsilon_2\,\epsilon_3\cdot\epsilon_4 - \epsilon_1\cdot\epsilon_3\,\epsilon_2\cdot\epsilon_4\right)$$



$$i\mathcal{M} = \epsilon_1^{\alpha}\epsilon_2^{\beta}\epsilon_3^{\rho *}\epsilon_4^{\sigma *}\cdot(igm_W\, g_{\alpha\beta})\,(igm_W\, g_{\rho\sigma})\cdot\frac{i}{(p_1+p_2)-m_h^2}$$

$$= -i\,\epsilon_1\cdot\epsilon_2\,\epsilon_3^*\cdot\epsilon_4^*\,\frac{g^2 m_W^2}{s-m_h^2}$$



$$i\mathcal{M} = \epsilon_1^{\alpha}\epsilon_2^{\beta}\epsilon_3^{\rho *}\epsilon_4^{\sigma *}\,(igm_W\, g_{\alpha\rho})\,(igm_W\, g_{\beta\sigma})\,\frac{i}{(p_1-p_3)^2-m_h^2}$$

$$= -i\,\epsilon_1\cdot\epsilon_3^*\,\epsilon_2\cdot\epsilon_4^*\,\frac{g^2 m_W^2}{t-m_h^2}$$