QFT3.

The Standard Model and EW Theory

Lecture 3.

November 11, 2020.

Last time: SM at tree-level

Today: More tree-level analysis of EWSB
- Unitarity & Higgsless theories
- Unitarity & fermion axial-vector couplings
- New gauge bosons.

Last time, we went through the SM at tree level. We concluded with the calculation of W and Z decay widths, which are dominated by tree-level couplings.

When analyzing W & Z interactions, it is useful to encode the interactions as the currents mediated by each vector. This is done simply by finding the coefficient of $W_\mu^{+/-}$ or Z_μ in each matter field covariant derivative:

$$\bar{\Psi}\, i \not{D}\, \Psi = \bar{\Psi}\, i\not{\partial}\, \Psi + \bar{\Psi}\frac{g}{\sqrt{2}}\left(\not{W}^{+} T^{+} + \not{W}^{-} T^{-}\right)\Psi$$
$$+ \bar{\Psi}\frac{1}{\sqrt{g^2+g'^2}} \not{Z}\left(g^2 T^3 - g'^2 Y\right)\Psi + \bar{\Psi}\, e A_\mu (T^3 + Y)\Psi$$

Using $c_W = \frac{g}{\sqrt{g^2+g'^2}}$, $s_W = \frac{g'}{\sqrt{g^2+g'^2}}$,

the Z coupling is $\bar{\Psi}\, \frac{g}{c_W} \not{Z}\, (T^3 - s_W^2 Q)$.

This is useful because the EOM of the vector fields will extract the current coupling to the matter fields, $\frac{\delta \mathcal{L}}{\delta V_\mu} = J_\mu$.

We get (P+S, eqn. 20.80).

$$\mathcal{L} \supset g\left(W_\mu^+ J_W^{\mu +} + W_\mu^- J_W^{\mu -} + Z_\mu^0 J_Z^\mu\right) + e A_\mu J_{EM}^\mu$$

$$J_W^{\mu +} = \frac{1}{\sqrt{2}}\left(\bar{\nu}_L \gamma^\mu e_L + \bar{u}_L \gamma^\mu d_L\right)$$

$$J_W^{\mu -} = \frac{1}{\sqrt{2}}\left(\bar{e}_L \gamma^\mu \nu_L + \bar{d}_L \gamma^\mu u_L\right)$$

$$J_Z^\mu = \frac{1}{c_W}\Big(\bar{\nu}_L \gamma^\mu \tfrac{1}{2}\nu_L + \bar{e}_L \gamma^\mu\left(-\tfrac{1}{2} + s_W^2\right)e_L + \bar{e}_R \gamma^\mu (s_W^2) e_R$$
$$+ \bar{u}_L \gamma^\mu \left(\tfrac{1}{2} - \tfrac{2}{3}s_W^2\right)u_L + \bar{u}_R \gamma^\mu\left(-\tfrac{2}{3}s_W^2\right)u_R$$
$$+ \bar{d}_L \gamma^\mu\left(-\tfrac{1}{2} + \tfrac{1}{3}s_W^2\right)d_L + \bar{d}_R \gamma^\mu \left(\tfrac{1}{3}s_W^2\right)d_R\Big)$$

$$J_{EM}^\mu = \bar{e}\gamma^\mu(-1)e + \bar{u}\gamma^\mu\left(\tfrac{2}{3}\right)u + \bar{d}\gamma^\mu\left(\tfrac{-1}{3}\right)d.$$

Given our expression for the decay widths, we can rewrite the currents using vector + axial vector couplings instead of LH + RH couplings.

Recall, $e_L \equiv P_L e = \left(\frac{1-\gamma_5}{2}\right)e$

$$J_W^{\mu +} = \frac{1}{\sqrt{2}}\left(\bar{\nu}\gamma^\mu\left(\tfrac{1}{2} - \tfrac{\gamma^5}{2}\right)e + \bar{u}\gamma^\mu\left(\tfrac{1}{2} - \tfrac{\gamma^5}{2}\right)d\right)$$

$J_W^{\mu -}$ = h.c. of $J_W^{\mu +}$

$$J_Z^\mu = \frac{1}{c_W}\Big(\bar{\nu}\gamma^\mu\left(\tfrac{1}{4} - \tfrac{\gamma^5}{4}\right)\nu + \bar{e}\gamma^\mu\left(-\tfrac{1}{4} + s_W^2 + \tfrac{1}{4}\gamma^5\right)e$$
$$+ \bar{u}\gamma^\mu\left(\tfrac{1}{4} - \tfrac{2}{3}s_W^2 - \tfrac{1}{4}\gamma^5\right)u + \bar{d}\gamma^\mu\left(\tfrac{-1}{4} + \tfrac{1}{3}s_W^2 + \tfrac{1}{4}\gamma^5\right)d\Big)$$

J_{EM}^μ = same as above.

Note patterns of +/- in J_Z^μ: upper components of doublets have $+\frac{1}{4}$ and $-s_W^2$ + $\frac{-1}{4}\gamma^5$, lower components have signs flipped.

It should be clear that the currents will simply have more terms when we add more matter fields with new $SU(2) \times U(1)$ representations. But keep in mind that $SU(2) \times U(1)$ is a chiral gauge theory with complete anomaly-free sets of chiral fermions.

Since the gauge coupling before each current is a free parameter, it seems easy to add new gauge interactions to the SM.

Ex. Add new W' interaction.

Here, we can add a new $U(1)'$ to the SM with gauge coupling g_X:

$$\mathcal{L} = g_X Z'_\mu J^\mu_{Z'} .$$

Certainly, as $g_X \to 0$, we would to recover the SM as a limiting case & thus we cannot exclude this possibility.

What kind of terms are possible for $J^\mu_{Z'}$?

Must retain EW symmetry structure & make Z' massive. Hence, vector-like couplings similar to J^μ_{EM} are easy to engineer, as long as we are orthogonal (in choosing charges) to the $SU(2) \times U(1)$ reps.

Ex. $B-L$. Quarks have charge $\frac{1}{3}$, leptons are -1.

$$\mathcal{L} = \frac{1}{3} g_X \bar{q} \gamma^\mu Z'_\mu q + -1 g_X \bar{l} \gamma^\mu Z'_\mu l - 1 g_X \bar{\nu} \gamma^\mu Z'_\mu \nu .$$

~~Since the fermions have vectorlike charge assignments~~

We get pure vector interactions after EWSB. Still, since SM fermions have chiral representations, there are non-trivial anomaly cancellation conditions for the theory to be consistent.

Ex. copy of $U(1)_{EM}$ with heavy A' photon. Usually from kinetic mixing.

We have looked briefly at how vector coupling can arise for SM fermions. But what about axial vector couplings?

The axial vector coupling is inherently a chiral coupling, so it is not a free parameter. In fact, it follows a tree-level relation relating the fermion mass and gauge coupling, dictated by perturbativity unitarity requirements.

Unitarity & EWSB.

Prior to the Higgs discovery, we had a "no-lose" theorem that the LHC (or any other collider at high enough energy) would produce a Higgs boson or other new phenomena (Ref. Lee, Quigg, Thacher, 1977).

The argument is based on understanding the scattering amplitude for massive gauge bosons as function of increasing center of mass energy.

First, recall that massless vector bosons have two degrees of freedom: two transverse polarizations, ϵ_+ + ϵ_-. (See P+S, p. 511)

A massive vector has three polarizations, $\epsilon_+, \epsilon_-, \epsilon_0$, where ϵ_0 is the longitudinal polarization. (See Schwartz, 8.2) + 24.1.5.

For $p^\mu = (E, 0, 0, p_z)$, we can choose

$$\epsilon^\mu_{T_1} = (0,1,0,0), \quad \epsilon^\mu_{T_2} = (0,0,1,0), \quad \epsilon^\mu_L = \frac{1}{m}(p_z, 0, 0, E)$$

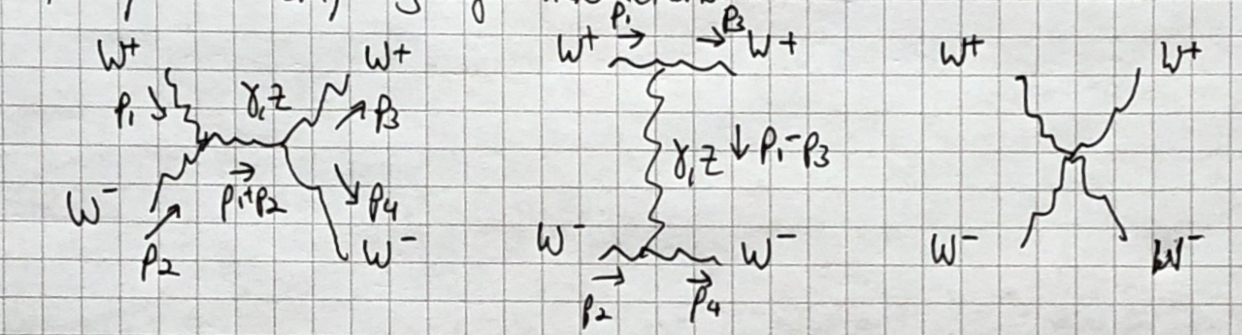
which satisfy $\epsilon \cdot p = 0$ + $\epsilon_i \cdot \epsilon_j^* = -\delta_{ij}$.

Now, calculate 2-to-2 scattering with EW gauge bosons.

There are several possible diagrams that show unitarity violation:

$W^+W^- \to W^+W^-$, $W^+W^- \to ZZ$, $W^+W^+ \to W^+W^+$, $ZZ \to ZZ$, $W^{\pm}Z \to W^{\pm}Z$

Let us analyze $W^+W^- \to W^+W^-$: Consider a Higgsless theory w/ only gauge interactions.



The matrix elements are written in Chang, Cheung, Lu, Yuan (1303.6335). [Ex: verify these expressions.]

$$i\mathcal{M}_t^{\gamma+Z} = -ig^2\left(\frac{s_W^2}{t} + \frac{c_W^2}{t-m_Z^2}\right)$$

$$\left[(p_1+p_3)^\mu \epsilon_1\cdot\epsilon_3^* - 2p_3\cdot\epsilon_1\,\epsilon_3^{\mu *} - 2p_1\cdot\epsilon_3^*\,\epsilon_1^\mu\right]$$
$$\left[(p_2+p_4)_\mu \epsilon_2\cdot\epsilon_4^* - 2p_4\cdot\epsilon_2\,\epsilon_{\mu 4}^* - 2p_2\cdot\epsilon_4^*\,\epsilon_{\mu 2}\right]$$

$$i\mathcal{M}_s^{\gamma+Z} = -ig^2\left(\frac{s_W^2}{s} + \frac{c_W^2}{s-m_Z^2}\right)$$

$$\left[(p_1-p_2)^\mu \epsilon_1\cdot\epsilon_2 + 2p_2\cdot\epsilon_1\,\epsilon_2^\mu - 2p_1\cdot\epsilon_2\,\epsilon_1^\mu\right]$$
$$\left[(p_4-p_3)_\mu \epsilon_3^*\cdot\epsilon_4^* - 2p_4\cdot\epsilon_3^*\,\epsilon_{\mu 4}^* + 2p_3\cdot\epsilon_4^*\,\epsilon_{\mu 3}^*\right]$$

$$i\mathcal{M}_4 = ig^2\left[2\epsilon_2\cdot\epsilon_3^*\,\epsilon_1\cdot\epsilon_4^* - \epsilon_2\cdot\epsilon_1\,\epsilon_3^*\cdot\epsilon_4^* - \epsilon_2\cdot\epsilon_4^*\,\epsilon_1\cdot\epsilon_3^*\right]$$

We can, in the high-energy limit, approximate the polarization vectors by the leading longitudinal piece:

$$\epsilon_{1L}^\mu \approx \frac{p_1^\mu}{m_W} - \frac{2m_W}{s}p_2^\mu$$

$$\epsilon_{2L}^\mu \approx \frac{p_2^\mu}{m_W} - \frac{2m_W}{s}p_1^\mu$$

Note $\epsilon_1\cdot p_1 \neq 0$ anymore.

Continue in Lec. 4.