Lecture 1, November 4, 2020

Wednesday, November 4, 2020 2:02 PM

Welcome

Administrivia

Motivation for QFT 3:

In general, QFT 1 + 2 teach nuts + bolts of QFT, but they are very light on real world calc.

At end of QFT 2, see SM + EW theory, but typically very briefly.

As a practicing particle physicist, though, SM predictions should be the main emphasis.

We know SM is not correct

- missing DM, DE, θ-QCD, nature of neutrinos, baryon asymmetry, quantum gravity...

My attitude: Must know QFT of SM inside + out.

Two approaches to proving SM is wrong + why both approaches necessitate in depth study of SM.

1.) Make precision measurements of SM observable.

Hope for discrepancy b/t precise theory calc. + experimental result of SM observable will invalidate SM.

2.) Make some arbitrary measurement with some fantastic apparatus, hopefully see something spectacular that is an unambiguous signal.

This necessarily tries to define the SM by its complement.

We have to know (+ calculate) what SM <u>predicts</u> + <u>does</u> <u>not</u> <u>predict</u>.

Second main motivation for this course:

QFT is hard!!

Imagine you wanted to test QFT.

In essence, every test of QFT inescapably relies on under SM interactions + particles.

In classical physics (+ even in QM), possible to prepare distinct systems to test the equations of classical physics.

All knowledge we have about QFT (+ that it is right) is done from the lens of SM.

Last comment:

In general, it is very useful to calculate in SM without "opening the box," e.g. w/o inputting <u>any</u> SM parameters (masses or couplings).

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Goal: teaches you to think of families of Lagrangians that share same structure + differ only in couplings + masses.

Distinguish b/t "our SM" (the SM) vs. "a SM."

Example: problem of baryogenesis vs. dark matter.

<u>Return: 3:10 pm</u>

Review: SM symmetries and field content

SM is $SU(3)_c \times SU(2)_w \times U(1)_Y$ gauge symmetries

$SU(2)_w \times U(1)_Y$ are chiral.

Field content:

3 flavors of each
$$\left\{\begin{array}{l} Q_L \sim (3, 2, 1/6) \\ u_R \sim (3, 1, 2/3) \\ d_R \sim (3, 1, -1/3) \end{array}\right. \qquad \begin{array}{l} L_L \sim (1, 2, -1/2) \\ e_R \sim (1, 1, -1) \\ [N_R \sim (1, 1, 0)] \end{array} \qquad [\, Q = T_3 + Y \,]$$

$$H \sim (1, 2, 1/2)$$

Since all fermions are chiral, bare mass terms for fermions are forbidden.

Invoke Yukawa couplings + EWSB to generate masses

[illegible]

Invoke Yukawa couplings + EWSB to generate masses for all fundamental fermions.

EWSB [spont. symmetry breaking of EW sym.] is achieved via Higgs potential

$$V(H) = -\mu^2 |H|^2 + \lambda |H|^4$$

$\mu^2 > 0$ to trigger EWSB.

SM Lagrangian:

$$\mathcal{L} = \mathcal{L}_{gauge} + \mathcal{L}_{\cancel{CP}} + \mathcal{L}_{Kin} + \mathcal{L}_{Yukawa} + \mathcal{L}_{Higgs} \; [+ \mathcal{L}_{Maj}]$$

$$\mathcal{L}_{gauge} = -\frac{1}{4} G^{\mu\nu,a} G_{\mu\nu}^{a} - \frac{1}{4} W_{\mu\nu}^{i} W^{i\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

$a = 1,..8 \qquad i = 1,..3$

$$\mathcal{L}_{\cancel{CP}} = \frac{\theta g_s^2}{32\pi^2} G_{\mu\nu}^{a} \tilde{G}^{\mu\nu,a} + \frac{\theta_2}{32\pi^2} g^2 W_{\mu\nu}^{i} \tilde{W}^{i\mu\nu} + \frac{\theta_1 g'^2}{16\pi^2} B_{\mu\nu} \tilde{B}^{\mu\nu}$$

$$\mathcal{L}_{Kin} = \bar{Q}_L^j i\cancel{D} Q_L^j + \bar{u}_R^j i\cancel{D} u_R^j + \bar{d}_R^j i\cancel{D} d_R^j$$
$$+ \bar{L}_L^j i\cancel{D} L_L^j + \bar{e}_R^j i\cancel{D} e_R^j \; [+ \bar{N} i\cancel{\partial} N]$$

$j = 1,...3$ flavor index

$$\mathcal{L}_{Yuk} = -y_u^{ij} \bar{Q}_L^i \tilde{H} u_R^j - y_d^{ij} \bar{Q}_L^i H d_R^j - y_e^{ij} \bar{L}_L^i H e_R^j + h.c.$$
$$(- y_\nu^{ij} \bar{L}_L^i \tilde{H} N^j + h.c.)$$

$$\tilde{H} \equiv i\sigma_2 H^*$$

$$\mathcal{L}_{Higgs} = |D_\mu H|^2 - V(H)$$

$$\left[\mathcal{L}_{Maj} = -m_N \bar{N}_i^c N_i \right]$$

Minimize Higgs potential, expand around vev.

$$v^2 = -\frac{\mu^2}{\lambda} \qquad \left(\begin{array}{l} v = 246 \text{ GeV}) \\ H \sim \frac{1}{\sqrt{2}}(v+h) \end{array} \right)$$

Effect is massive EW gauge bosons + SM charged fermions.

Calculate EWSB:

$$D_\mu H \supset -ig\frac{\tau^i}{\sqrt{2}} W_\mu^i \begin{pmatrix} 0 \\ v \end{pmatrix} \boxed{-\frac{ig'}{2\sqrt{2}} B_\mu \begin{pmatrix} 0 \\ v \end{pmatrix}}$$

$$\tau^i W_\mu^i = \frac{1}{2}\left(\sigma^1 W_\mu^1 + \sigma^2 W_\mu^2 + \sigma^3 W_\mu^3 \right)$$

$$= \frac{1}{2}\begin{pmatrix} W_\mu^3 & W_\mu^1 - iW_\mu^2 \\ W_\mu^1 + iW_\mu^2 & -W_\mu^3 \end{pmatrix}$$

$$D_\mu H \supset \begin{pmatrix} \frac{1}{2\sqrt{2}} (-ig)(W_\mu^1 - iW_\mu^2) v \\ \frac{1}{2\sqrt{2}} (-ig)(-W_\mu^3) v - \frac{ig'}{2\sqrt{2}} B_\mu v \end{pmatrix}$$

$$|D_\mu H|^2 \Big|_{v^2} = \frac{1}{8}\left(g^2 \boxed{v^2} (W_\mu^1 - iW_\mu^2)(W_\mu^1 + iW_\mu^2) \right. + \frac{1}{8} \boxed{v^2} \cdot (-g W_\mu^3 + \cancel{g' B_\mu})^2$$

These define the mass eigenstates for $W^{+/-}$, Z, A.

$$W^+_\mu = \frac{1}{\sqrt{2}}\left(W^1_\mu - iW^2_\mu\right) \quad , \quad W^-_\mu = (W^+_\mu)^*$$

$$Z_\mu = \frac{1}{\sqrt{g^2+g'^2}}\left(-gW^3_\mu + g' B_\mu\right)$$

$$A_\mu = \frac{1}{\sqrt{g^2+g'^2}}\left(g' W^3_\mu + g B_\mu\right)$$

$$\Rightarrow \quad |D_\mu H|^2\Big|_{v^2} = \frac{1}{4} g^2 v^2 W^+_\mu W^-_\mu + \frac{(g^2+g'^2)}{8} v^2 Z_\mu Z^\mu$$

$$\equiv m_W^2 W^+_\mu W^{-\mu} + \frac{1}{2} m_Z^2 Z_\mu Z^\mu$$

for $m_W^2 = \frac{g^2 v^2}{4}$, $\quad m_Z^2 = \frac{(g^2+g'^2)}{4} v^2$

In this new basis, couplings to photon always prop. to $e \equiv g s_W$, $\quad s_W \equiv \frac{g'}{\sqrt{g^2+g'^2}}$, $\quad c_W = \frac{g}{\sqrt{g^2+g'^2}}$.

For arb. rep. of $SU(2) \times U(1)$:

$$D_\mu = \partial_\mu - \frac{ig}{\sqrt{2}}\left(W^+_\mu T^+ + W^-_\mu T^-\right) - \frac{ig}{c_W} Z_\mu\left(T^3 - s_W^2 Q\right)$$
$$- ie A_\mu Q$$

$[Q = T^3 + Y]$

Thought experiment:

What if $g' \to 0$?

Break $SU(2)$ completely, photon aligned purely with $U(1)_Y$ & no coupling.