Lecture 16, January 27, 2021

Wednesday, January 27, 2021 9:54 AM

Last time: EW oblique parameters

Today: dim. 6 SM EFT

DM EFT + simplified models

EW oblique parameters were motivated because you could directly test form factors for EW gauge bosons at specific choices of $Q^2 = 0$ or m_Z^2.

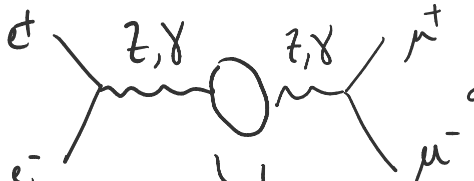
$\Rightarrow$ During the time of LEP: use e^+e^- at $\sqrt{s} = m_Z$

Needed to characterize new physics corrections at this specific Q^2.

Roughly $T \sim \Pi_{WW}(0) - \Pi_{ZZ}(0)$

$S \sim \Pi_{ZZ}(0) - \Pi_{\gamma\gamma}(m_Z^2) + \ldots$

Initial state $\sqrt{s} = m_Z$



or other final state

can constrain possible heavy new degrees of freedom in loop.

Now, with LHC + pp collider, we don't have precision control over momentum transfer. We still want to test NP through form factors (i.e. finite Q^2 behavior of gauge bosons) $\Rightarrow$ study all possible Q^2 via differential distributions.

Discard EW oblique observables $\Rightarrow$ subsume into dim-6 SM EFT.

Any EFT is defined by set of dynamical, light degrees of freedom, gauge symmetries + possibly global, + truncation of higher dim. operators.

Canonical example:

$e^\mp, \mu^\mp, \overset{(-)}{\nu}_e, \overset{(-)}{\nu}_\mu$,

$\Rightarrow$ kinetic + mass is easy + QED

$\Rightarrow$ dim.-6 G_F interaction: $\frac{1}{\Lambda^2}\bar e \gamma_\mu P_L \nu_e \, \bar\mu \gamma^\mu P_L \nu_\mu$.

Could have added dim-6 interactions

$$\frac{1}{\Lambda^2}\bar e \gamma_\mu P_L \nu_e \, \bar e \gamma^\mu P_L \nu_e \quad + \quad e \leftrightarrow \mu$$

violate charge $\times$ $\frac{1}{\Lambda^2}\bar \nu_e \gamma_\mu P_L \nu_e \, \bar e \gamma^\mu P_L \nu_e$

$$\frac{1}{\Lambda^2}\bar \nu_e \gamma_\mu P_L \nu_e \, \bar \nu_e \gamma^\mu P_L \nu_e \quad + \quad e \leftrightarrow \mu$$

$$\frac{1}{\Lambda^2}\bar \nu_\mu \gamma_\mu P_L \nu_\mu \, \bar \nu_e \gamma^\mu P_L \nu_e$$

$\star$ $\frac{1}{\Lambda^2}\bar e \gamma_\mu P_L e \, \bar\mu \gamma^\mu P_L \mu$ + 4e or 4μ

$\frac{1}{\Lambda^2}\bar e \gamma_\mu P_L \mu \, \bar\mu \gamma^\mu P_L \mu$ or 3e/μ or 3μ/e

Also add $P_L \to P_R$. (orthogonal operator)

Pure scalar + pseudoscalar

$\frac{1}{\Lambda^2}\bar e \nu_e \bar\mu \nu_\mu \leftarrow$

$\star$ $\frac{1}{\Lambda^2}(\bar e e)(\bar\mu \mu)$ or $\frac{1}{\Lambda^2}(\bar e \gamma_5 e)(\bar \mu \mu)$ or $\frac{1}{\Lambda^2}(\bar e \gamma_5 e)(\bar\mu \gamma^5 \mu)$

$$\{\gamma_\mu, \gamma_\mu \gamma_5\} \to \{\gamma_\mu P_L, \gamma_\mu P_R\}$$

$\{\gamma_\mu, \gamma_\mu\gamma_5\}, \{\gamma_\mu P_L, \gamma_\mu P_R\}$

$$\frac{1}{\Lambda^2} \bar{e} \sigma^{\mu\nu} e \; \bar{\mu} \sigma_{\mu\nu} \mu$$

Recall: Lorentz covariance: 5 basic fermion bilinears.

$$\bar{\psi}\psi, \quad \bar{\psi}\gamma_5\psi, \quad \bar{\psi}\gamma^\mu\psi, \quad \bar{\psi}\gamma^\mu\gamma^5\psi, \quad \bar{\psi}\sigma^{\mu\nu}\psi.$$

$$1 \qquad 1 \qquad 4 \qquad 4 \qquad 6$$

16 generators. Matches Dirac algebra.

Distinct from top-down construction.

W boson, $m_W^2 = (80.4)^2\,\mathrm{GeV}^2$, want EFT for $Q^2 \ll m_W^2$,

Know the UV theory, integrate out W boson.

EOM of W boson + $G_F = J_{W^+} \otimes J_{W^-}$.

<u>Not</u> generate $\frac{1}{\Lambda^2}\bar{e}e\bar{\mu}\mu$.

But from bottom-up approach: don't know UV physics, only constraint is to stop at dim. 6, then must include $\frac{1}{\Lambda^2}\bar{e}e\bar{\mu}\mu + \frac{1}{\Lambda^2}\bar{e}\gamma^\mu P_L \nu_e \bar{\mu}\gamma_\mu P_L \nu_\mu$.

So, scattering experiments with these final states will distinguish different operator structures through angular correlations & distributions.

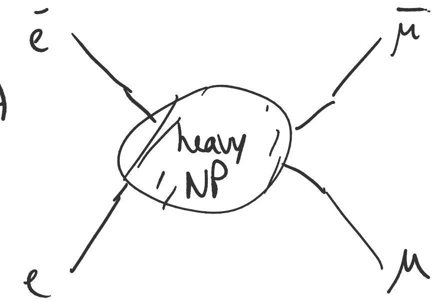
From bottom-up: write <u>all</u> possible operators.

Superset of UV completions of <u>all</u> operators will be very complicated.

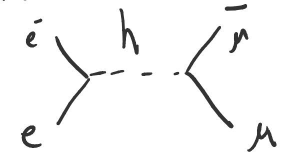
From top-down: write operators that correspond to given UV completion.

UV competion of given operator is usually easy to construct. Split operator into constituent tree-level vertices with some mediator.

Ex. $\left[\frac{1}{\Lambda^2}(\bar{e}e)(\bar{\mu}\mu)\right. \ni$

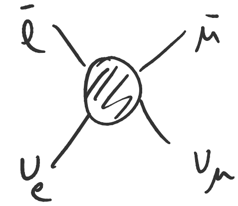


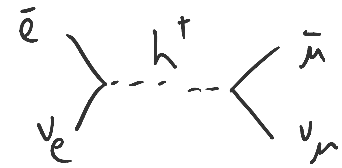
Resolves to



So, UV completion is SM Higgs.

$\frac{1}{\Lambda^2}(\bar{e}\,\nu_e)\,(\bar{\mu}\,\nu_\mu) \ni$





UV completion is charged Higgs.

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① Easy to construct EFT from UV, simply because match dofs.

② Distinction between EFT from SM vs. SM EFT.

First is using SM as UV theory, + study at lower energies. Integrate out W, Z, h, top quark. Useful for isolating the relevant dofs of an exp. at low energies.

Top-down

Second, (SM EFT) consider SM as the low energy dofs of arbitrary UV completion. Build a generic dim-6 interacting theory from SM fields.

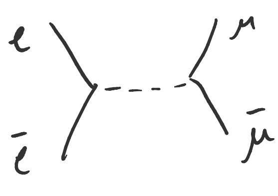
Bottom-up

③ Easy (usually) to construct UV from EFT; i.e. given

③ Easy (usually) to construct UV from EFT; i.e. given any operator in SM EFT, view it as some effective diagram of SM fields as external states & draw tree-level (usually) heavy particle in middle.

④ EFTs are generally renormalizable, even though they include non-ren. ops. ⇒ Renormalizability is instead a cutoff on scales where EFT description is valid.

Distinction b/t NP is tree vs. loop



$$\Rightarrow \underbrace{\frac{1}{p^2 - m_h^2}}_{\text{dim.-6 op.}} \sim \frac{-1}{m_h^2}\Big(1 + \underbrace{\frac{p^2}{m_h^2}}_{\text{dim.-8}} + \underbrace{\cdots}_{\text{dim.-10},\ldots}\Big)$$

$$\underbrace{\frac{-1}{m_h^2}(1)}$$

$\bar{e} h e + \bar{\mu} h \mu$

$$\Rightarrow \underbrace{\frac{1}{m_h^2}\bar{e}e\,\bar{\mu}\mu}_{\substack{\text{usually loop}\\ \text{correction} \sim \text{dim.-8}}} + \underbrace{\frac{1}{m_h^4}(\bar{e}\partial_\mu e)(\bar{\mu}\gamma^\mu\mu)}_{\substack{\text{intrinsic correction}\\ \text{from EFT.}}} + \frac{1}{m_h^6}\cdots$$

Top-down.

$$\mathcal{L}_{UV} = \mathcal{L}_4 + \mathcal{L}_6 + \cancel{\times}$$

$$\mathcal{L}_{UV}^{ren.} = \mathcal{L}_{UV}^{bare} + \mathcal{L}_{UV}^{CT} + \mathcal{L}_{UV}^{1\text{-loop}}$$

$$= \mathcal{L}_{UV,L}^{bare} + \mathcal{L}_{UV,H}^{bare} + \mathcal{L}_{UV,L}^{CT} + \mathcal{L}_{UV,H}^{CT} + \mathcal{L}_{UV,L}^{1\text{-loop}} + \mathcal{L}_{UV,H}^{1\text{-loop}}$$

$$= \mathcal{L}^{\text{"}\mathcal{L}_4\text{"}} + \mathcal{L}^{bare} + \mathcal{L}_{UV,H}^{CT} + \mathcal{L}_{UV,H}^{1\text{-loop}} \quad \Big] \text{ UV div. at 1-loop}$$

$= \overset{\text{"}\mathcal{L}_4\text{"}}{\mathcal{L}_{EFT}} + \mathcal{L}^{bare}_{UV,H} \cancel{+ \mathcal{L}^{CT}_{UV,H} + \mathcal{L}^{1\text{-loop}}_{UV,H}} \Big]$ UV div. at 1-loop

can drop — is necessarily

$\downarrow$

no distinction in $\mathcal{L}^{bare}_{UV,H}$ vs. $\mathcal{L}^{ren}_{UV,H}$ if $\underbrace{\Lambda_{UV} \gg \Lambda^{cutoff}_{EFT}}$

all couplings + fields are renormalized.

$\downarrow$

Match $\mathcal{L}^{bare}_{UV,H}$ to $\mathcal{L}_4$

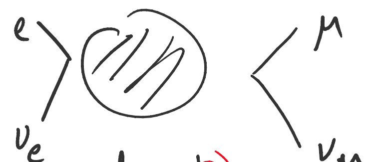
$\mathcal{L}_{EFT} = \mathcal{L}_4 + \mathcal{L}_6 + \dots$ has $\Lambda^{cutoff}_{EFT} \ll \Lambda_{UV}$

Easiest to estimate Λ^{cutoff}_{EFT} by asking when dim-8 is important.

$$\Rightarrow \frac{\overset{\text{exp. choice}}{p^2}}{\underbrace{\Lambda^2}_{\text{same scale as } \frac{1}{\Lambda^2} \text{ in dim. 6.}}} \sim 1 .$$

[How to renormalize dim.-6 EFT?

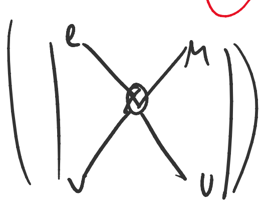
Perform scattering exp., test G_F.



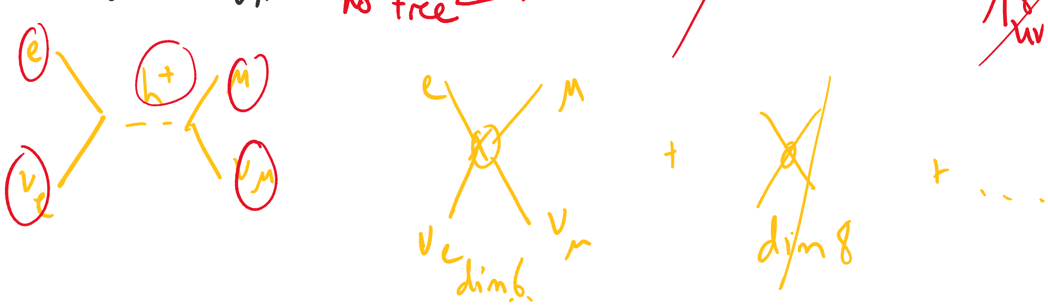
$\mathcal{L}_4 + \mathcal{L}_6 + \mathcal{L}_H$

only int. is dim-6

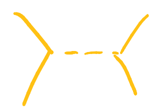
$$\Rightarrow \sigma \propto \left| \right|^2 \sim \underbrace{\frac{1}{\Lambda^4_{UV}}}_{\text{no tree}} + \dots \cancel{\frac{1}{\Lambda^6_{UV}}} \quad \cancel{\frac{1}{\Lambda^8_{UV}}}$$



dim-6-8 int + dim.-8 are truncated



$\Rightarrow \sigma \propto$

$\Rightarrow \sigma \propto \Big|$  $\Big|^2 + \mathcal{M}(\text{tree} \cdot \text{dim}6^*) + \mathcal{M}(\text{tree}^* \cdot \text{dim}6)$

ν_e dim6 μ aym 8

$+ |\mathcal{M}(\text{dim}8)|^2$

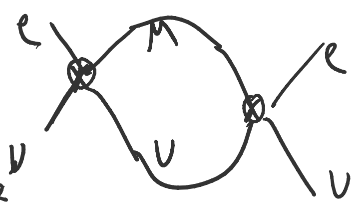
$$\sim \underbrace{\text{tree}^2 +}_{\text{keep}} \underbrace{\boxed{\frac{1}{\Lambda^2}} (\text{tree-dim.6 int.})}_{\text{keep} \checkmark} + \cancel{\frac{1}{\Lambda^4}(\text{dim.-6})^2}$$

drop, since tree-6 is more important

already absent, since dim.8 truncated

$$\Big[+ \frac{1}{\Lambda^4}(\text{tree-dim 8 int.}) + \frac{1}{\Lambda^6}(\text{dim}6\text{-}8) + \frac{1}{\Lambda^8}|\text{dim}8|^2$$

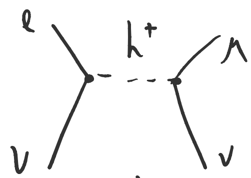
$$\int_0^\Lambda d^4k \frac{k}{(k^2)^2}$$

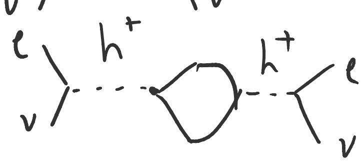


$\hookrightarrow \frac{C_6}{\Lambda^2} \cdot \frac{C_6}{\cancel{\Lambda^2}} \cdot \cancel{\Lambda^2}$

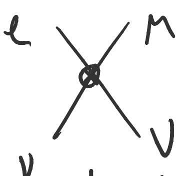
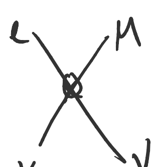
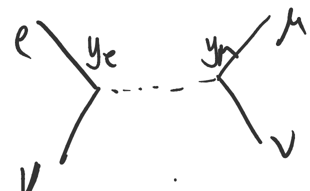
$= \delta C_6 \sim \frac{1}{\Lambda^2}$

$\mathcal{L}_{UV}$: h^+ renormalizable





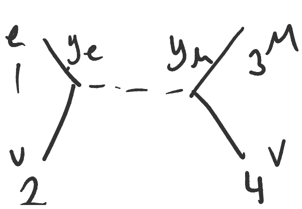
$\mathcal{L}_{EFT}$ at 1-loop.

 $+$  $+ \ldots =$ 

$$\underline{\frac{i}{m_h^2} y_e y_\mu + \frac{i y_e y_\mu (p_e \cdot p_\mu)}{m_h^4} + \ldots}$$

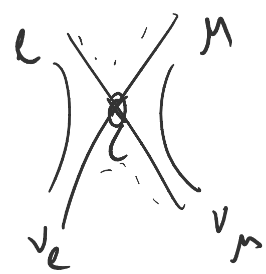
Match order by order in $\frac{1}{m_{h^+}^2}$

$$\frac{-i}{p^2 - m_{h^+}^2} \approx \frac{i}{m_h^2}\left(\underline{1} + \frac{p^2}{m_h^2} + \frac{p^4}{m_h^4} + \ldots\right)$$

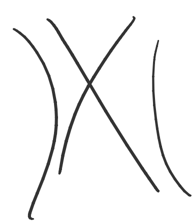


$$i\mathcal{M} = \bar{u}_4 \cdot i y_\mu \cdot v_3 \cdot \frac{-i}{p^2 - m_h^2} \cdot \bar{v}_2 \cdot i y_e \cdot u_1$$

$$= \frac{i y_\mu y_e}{p^2 - m_h^2} \left(\bar{u}_4^\alpha v_3^\alpha\right)\left(\bar{v}_2^\beta u_1^\beta\right)$$



$$i\mathcal{M}_6 = \frac{i C_6}{\Lambda^2} \cdot \left(\bar{u}_4 v_3\right)\left(\bar{v}_2 u_1\right)$$



$$i\mathcal{M}_8 = \frac{i C_8}{\Lambda^4} \left(\bar{u}_4 v_3\right)\left(\bar{v}_2 u_1\right)$$

Match $C_6, C_8, \ldots$ to Taylor series in

$$y_e y_\mu \left(\frac{1}{p^2 - m_h^2}\right) : -y_\mu y_e \frac{1}{m_h^2}\left(1 + \frac{p^2}{m_h^2} + \frac{p^4}{m_h^4} + \ldots\right)$$

UV description is <u>equivalent</u> to replacing scalar by $C_6, C_8, C_{10}, \ldots$ for this interaction

$$C_6 \equiv -\frac{y_\mu y_e}{m_h^2}, \quad C_8 \equiv -\frac{y_\mu y_e}{m_h^2}\left(\frac{p^2}{m_h^2}\right)$$

$\frac{p^2}{m_h^2}$ is expansion parameter in Taylor series.

If small, can approx. by 1st term, i.e. only C_6.