Lecture 14, January 20, 2021

Wednesday, January 20, 2021 2:15 PM

Course evaluations live (see email)

HW 5 also posted

Today: Move away from collider physics + PDFs

Open floor to suggested topics

Definitely: EW precision + Peskin-Takeuchi S, T, U + dim-6 SM EFT;

FCNCs in quarks/mesons, FCNCs in leptons;

CPV in SM quarks + leptons;

ν-physics - basic seesaw + variants, overview of expts.

Scattershot: Topics.

** Gauge unification; Higgs + DM; beam physics; CKM measurements; double Higgs; axion/U(1) counting; more frontier measurements or analyses @ colliders;

Factorization

** Dark model physics +/ axion

→ Rigorous treatment of DM existence.

→ Simplified models

* Hierarchy problem

From beginning, we talked through + calculated a lot of aspects of SM @ tree-level.

Now, try to understand SM @ 1-loop.

Already done for aspects of Higgs physics.

[Tree-level for Higgs physics is completely inadequate, need ggF & $h \to \gamma\gamma$ @ 1-loop.]

Categorize: how "distinct" is SM pheno @ 1-loop vs. tree level?

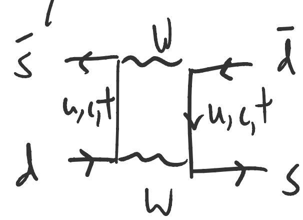
In other words, when does 1-loop diagram correct a tree-level diagram vs. give a qualitatively new amplitude/ operator?

☆ FCNCs!

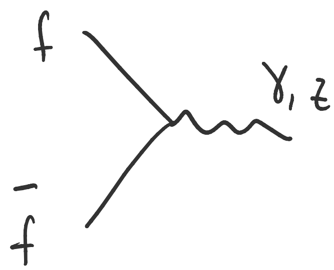
γ, Z . tree-level flavor conserving.

↳ Need a W loop.

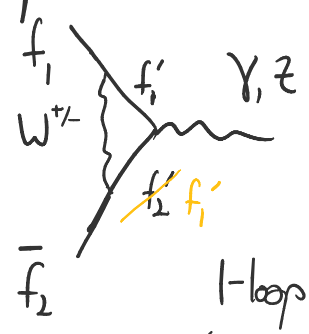
Diagramatically:



How do FCNCs arise @ 1-loop?



vs.

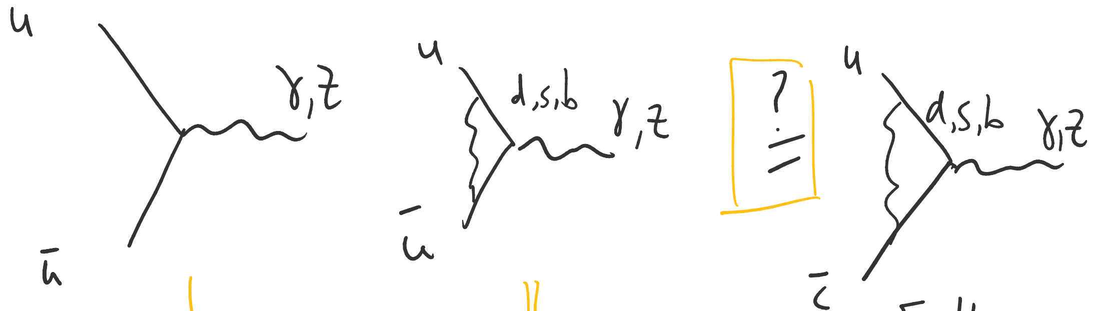


1-loop correction to γ, Z from W

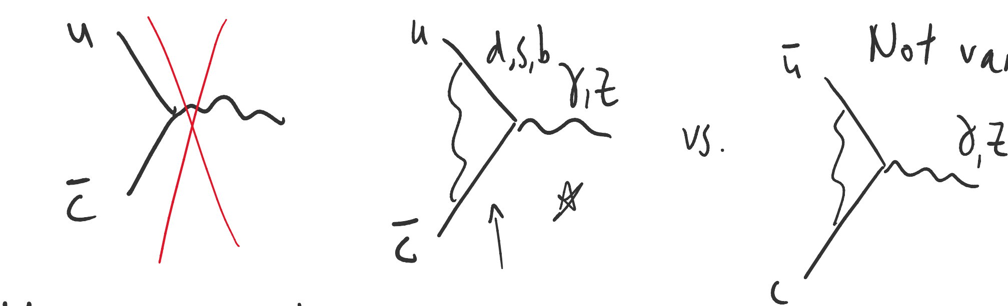
The intermediate vertex w/

...rection to γ, Z from W

The intermediate vertex w/ $f_1' \bar{f}_2' \gamma$ must be diagonal $\Rightarrow f_1' = f_2'$



Is this vanishing?



Not vanishing.

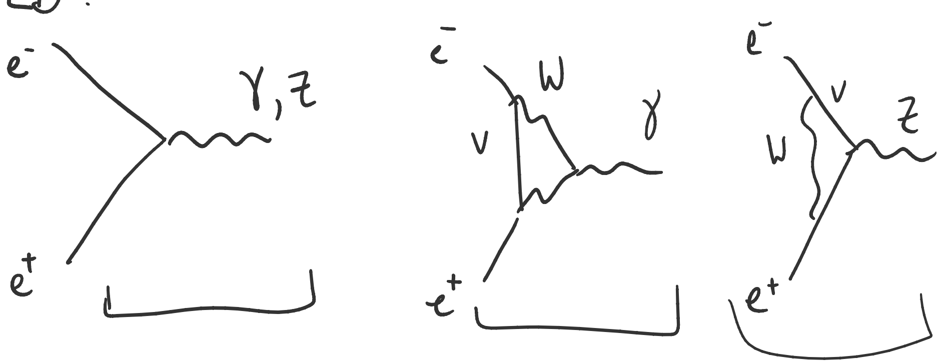
SM is renormalizable.

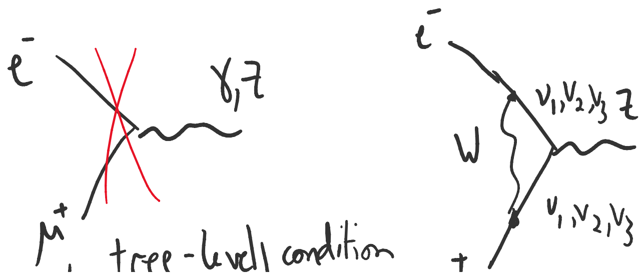
One-loop diagrams are generally divergent.

If there is no counterterm @ tree level, the divergence must vanish exactly.

↳ unitarity of CKM is very relevant.

QED:





tree-level condition

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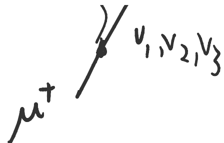
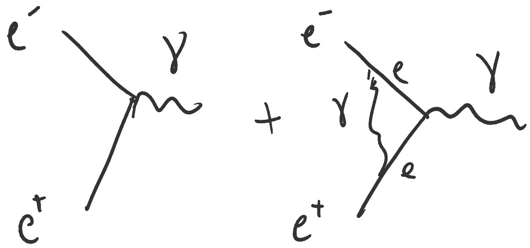


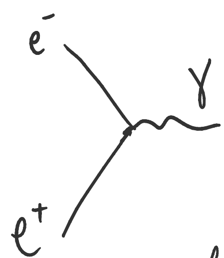
Answer in QED : vtx correction to $e^+e^-\gamma$:

$$\mathcal{L} \supset \bar{e} A_\mu \Big(\underbrace{e\gamma_\mu \cdot F_1(p^2)}_{\substack{\downarrow \\ \text{charge} \\ \text{renormalization} \\ \text{form factor}}} + \underbrace{q^\nu \sigma^{\mu\nu} F_2(p^2)}_{\substack{\downarrow \\ \text{magnetic} \\ \text{moment}}} \Big) e$$



Ren. cond. to eliminate UV divergence.

$\overline{MS}$:



@ $(q \to 0) \Rightarrow$ Acts with $e_R = e$ strength

wanted vtx to be exactly

$\mathcal{L} \supset e_R A_\mu \bar{e} \underbrace{\gamma^\mu} e$, pure γ^μ structure with e_R renormalized charge.

No possibility of generating $e^-\mu^+\gamma$ vtx in QED.

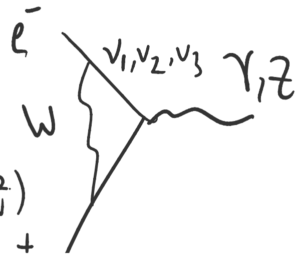
The tree-level condition : $F_1(q^2)$, $F_2(q^2) \to 0$ at $q^2 \to 0$ for $e^-\mu^+\gamma$ or $e^+\mu^-\gamma$ vtx.

So the loop is non-vanishing but only for finite q^2.

This necessarily need a symmetry reason / basis invariant argument for vanishing of UV divergence.

$\sum_i V_{ei} V^*_{\mu i}$

$\int \frac{d^4k}{(2\pi)^4} \frac{g \cdot (\not k + m_i)(\not k + m_i)}{(k^2 - m_i^2)^2 (k^2 - m_W^2)}$

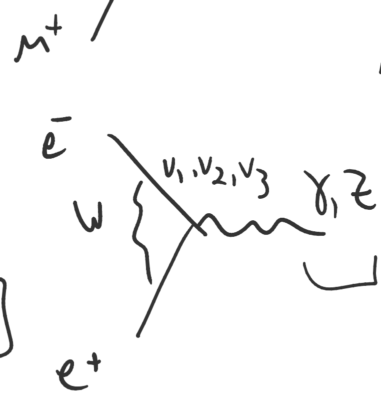


$$\underbrace{\left[\int dx\, dy\, dz \left(\frac{1}{\epsilon} \right) \cdot \right]}_{F(q^2, m_i, m_W)} \sum_i V_{ei} V^*_{\mu i} = 0 \text{ by unitarity of}$$

$\int \frac{d^4k}{(2\pi)^4} \frac{}{(k^2-m_i^2)^2\,(k^2-m_W^2)}$

$\frac{k^2}{k^6 - \Delta(m_i)}$

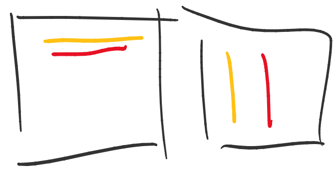
$V_{ei} V_{ei}^* g^3 \left[\frac{1}{\epsilon} - \log \Delta\right]$



$F(q^2, m_i, m_W)$
No need for CT

$\overset{\shortparallel}{0}$ by unitarity of PMNS.

$\left[\int dx\,dy\,dz \left(\frac{1}{\epsilon}\right)\right] \sum_i V_{ei} V_{ei}^*$
$\qquad\qquad\qquad\qquad\qquad\quad \shortparallel$
$\qquad\qquad\qquad\qquad\qquad\quad 1$



$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Need to have a CT.

for flavor-conserving coupling.

Exactly repeated for up + charm.

Punchline: ① 1-loop div. for any FV coupling vanishes by CKM unitarity.

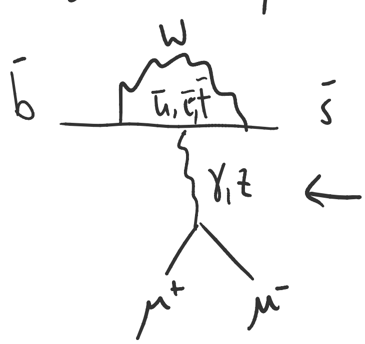
② FV only occurs for finite form factors.

Return @ 3:30pm

Not actually calculate any 1-loop vertex.

Will focus on the finite q^2-dependence in form factor.

Augment any vertex with subsequent γ/Z interaction.



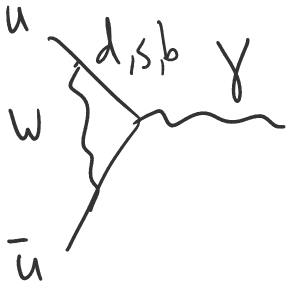
← changing γ/Z to propagator makes q^2-dependence explicit in amplitude.

This is called a penguin diagram.

Conveys the q^2-dependent FCNC for four-fermion interaction.

… the q^2-dependent FCNC for four-fermion interaction.

Can always take penguin diagram & cut on γ/Z leg to get $\underbrace{\text{1-loop vtx}}_{\text{flavor violating}}$ corrections we mentioned before.



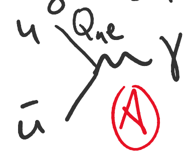
Generic 1-loop to $u\bar{u}\gamma$ coupling.

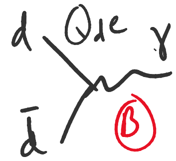
Involves g^2, e, $Q_d = -\frac{1}{3}$.

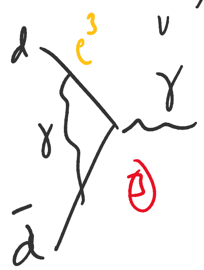
Q: What counterterm to associate to this divergence?

Answer: g, e @ $O(g, e, g^3, e^3, g^2 e, e^2 g)$.

1-gen. SM







Turn off g. ① + ③ survive, all else vanishes.

Know this is just QED renorm.

Restore g. Keep QED ren, but add $O(g^2)$ corr.

Define 1-loop CT for A & B

that satisfies 1,3, 2+4.

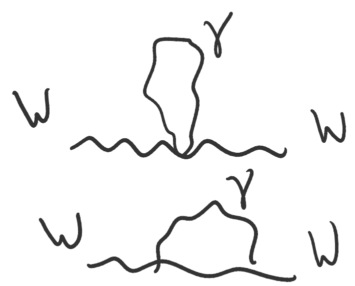
One CT for …

that satisfies 1,3, 2+4.

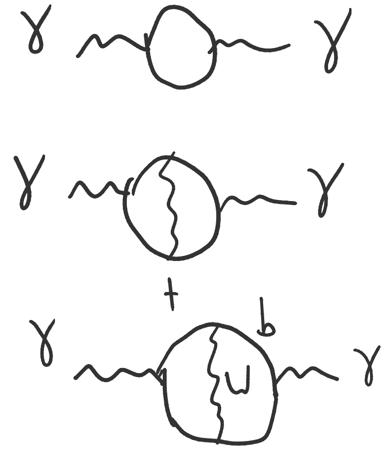
One CT for g that resolves 5+6.

Think of β-fcn. of $g \Rightarrow$ gets corrections at $\mathcal{O}(e^2)$.

β-fcn. of $e \Rightarrow$ gets corrections @ $\mathcal{O}(g^2)$.



$$\beta(g) \sim g + \frac{g^3}{16\pi^2} + \frac{g^5}{(16\pi^2)^2} \cdots$$



$$\beta(e) \sim e + e^3 + \ldots$$

tree + loop

$$e(g_s^2 + e^2 + 1)\frac{1}{\epsilon} \qquad g_s(g_s^2 + e^2 + 1)\frac{1}{\epsilon}$$

not independent

not independent

can both be solved by one set of CTs.

CTs at mixed order in couplings.