Lecture 14. ①

QFT 3: The Standard Model & EW Theory

Felix Yu

January 20, 2021

The SM @ 1-loop level.

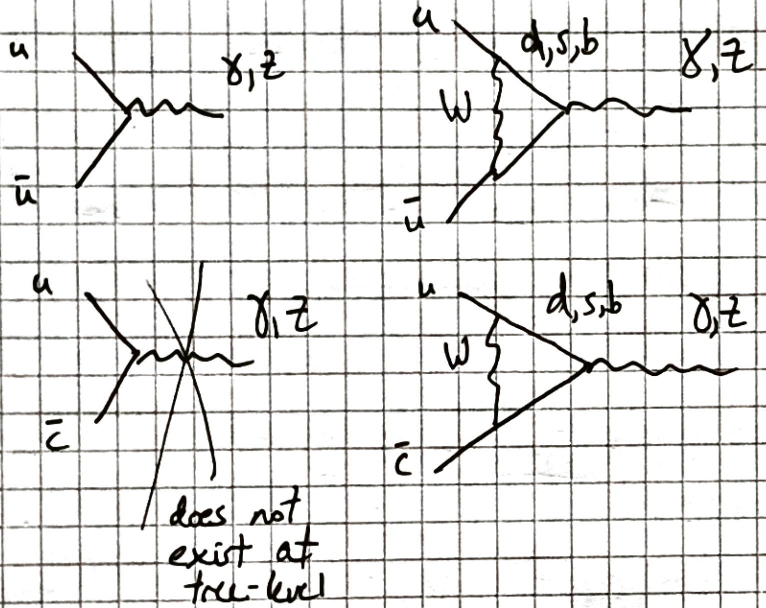
EW Precision Tests & SMEFT.

We have extensively discussed tree-level interactions in the SM with color, EW & QED interactions. The fermi interaction was central to understanding the dominant decays of K, D, & B mesons. The main aspect where tree-level physics fails, though, is regarding SM Higgs phenomenology. Because the quark couplings to the Higgs are proportional to mass, the tree-level production from $q\bar{q} \to h$ is very small. Instead, the loop-level $gg \to h$ & $h \to \gamma\gamma$ were critical production & decay interactions that in fact central reasons for the Higgs discovery in 2012. Other loop interactions that are new in Higgs physics include $h \to Z\gamma$ and the loop structures for operators of $h Z_{\mu\nu} Z^{\mu\nu}$ & $h W^+_{\mu\nu} W^{-\mu\nu}$.

When we study loop interactions in SM, we have to distinguish between when the loop diagrams gives rise to qualitatively new physics compared to a loop correction to an existing tree-level interaction.

The key type of interaction to consider are the flavor-conserving and flavor-violating interactions of fermions with gauge bosons.

For concreteness, consider the $f\bar{f}\gamma$ or $f\bar{f}Z$ vertex: We know that the SM global symmetry guarantees that the Higgs, γ, + Z (and gluon) couplings to fermions are all flavor diagonal at tree level. We can thus analyze the loop correction to the tree-level flavor conserving vertex + compare to the absent tree-level flavor-violating vertex:



Recall that these triangle loops are divergent generally.

$$\mathcal{M}_{loop} \sim \int \frac{d^4k}{(2\pi)^4} \frac{(k)^2}{(k^2-m_f^2)^2} \cdot \frac{1}{k^2} \sim \log \Lambda_{UV}.$$

Consequently, we needed to introduce a counterterm, corresponding to a tree-level bare coupling, that could absorb the counterterm.

In the flavor-conserving case, the counterterm calculation exists exactly as the QED case. The end result was

$$\mathcal{M}_{tree+1\text{-}loop} \propto \bar{u}\left(F_1(q^2)\gamma^\mu + F_2(q^2)\sigma^{\mu\nu}q_\nu\right)A_\mu f$$

where F_1 = electric form factor + F_2 = magnetic form factor.

Here, for the flavor-violating loop, there is no bare coupling that can absorb the divergence. So we must have another mechanism in order to get a finite answer.

It turns out that the 1-loop divergence for the flavor-violating loop cancels because of the unitary nature of the CKM matrix:

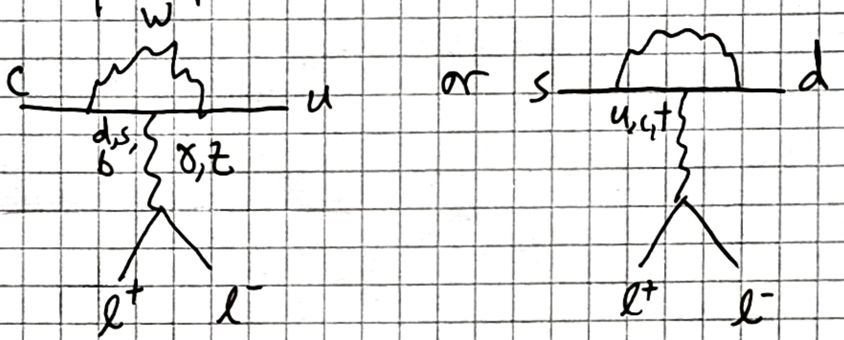
$$\mathcal{M}_{1\text{-loop}} \propto \left[\int \frac{d^4k}{(2\pi)^4} \frac{(\not{k})(\not{k})}{(k^2-m_i^2)^2\, k^2} \right] \sum_{i=d,s,b} V_{ui} V_{ci}^*$$

$$\sim \sum_{i=d,s,b} V_{ui} V_{ci}^* \left[(\log \Lambda_{UV}) \text{ or } \left(\frac{1}{\epsilon}\right) \right]$$

Since $\sum_{i=d,s,b} V_{ui} V_{ci}^* = 0$ as a unitary requirement on the CKM matrix, we actually do not need a counterterm. The corresponding finite form factors are necessary rations involving the quark masses + the W mass.

Now, what is the role of the renormalization condition? In QED, we need to insist that $F_1(q^2=0)=1$ + $F_2(q^2=0)=0$ to match to the classical long-range force behavior of electromagnetism. Here, given that the flavor-violating current has no tree-level interaction, it turns out that the γ or Z interaction will vanish for on-shell external vectors, which can also be understood as a consequence of the Ward identities. Hence, flavor-changing neutral currents are necessarily q^2-dependent.

The best way to understand the q^2-dependence of FCNCs is through the penguin diagram, which is a general class of 1-loop vertex corrections where the external γ / Z is connected to two further fermions (usually leptons). So we can take



Note that the γ / Z can also couple to the W-boson line.

In comparison to the FCNC vertex, the only difference is that the external γ / Z vector is now a propagator. So the $F_1(q^2)$ + $F_2(q^2)$ form factors are now transmitted to the dilepton interactions at finite q^2 through the $\frac{-i}{q^2 - m^2_{\gamma/Z}}$ propagator. In effect, the differential cross section w.r.t. q^2 is a direct test + probe of the form factors.