Lecture 12, January 7, 2021

Thursday, January 7, 2021 2:14 PM

Last time: Collider kinematics + dijet production at pp colliders

Today: Finish PDFs + IR singularities
KLN theorem
DGLAP equation

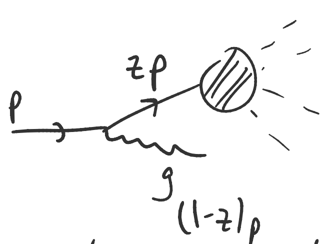
Kinoshita-Lee-Nauenberg: Any IR divergence from loop corrections is generally cancelled by phase space IR divergence. The SM is IR finite.

Technical details: short calculation in Schwartz, Sec. 32.2
(dim. reg. for ϵ) Explains cancellation $\frac{1}{\epsilon^2}$ IR divs. between virtual + real radiation and motivates remaining $\frac{1}{\epsilon}$ IR is absorbed by PDF evolution.

Following P+S, 17.5. (Parton evolution + splitting).

Starting point: initial electron in interaction
Consider the ISR-related diagram.

See G. Salam, 2009 QCD Lecture 2 on PDFs @ Bautzen



Hard process occurs after splitting, $p \to zp$.

$$\sigma_{g+h}(p) \simeq \sigma_h(zp) \cdot \frac{\alpha_s C_F}{\pi} \frac{dz}{1-z} \frac{dk_T^2}{k_T^2}$$

splitting fcn., will motivate today

$$\sigma_{V+h}(p) \simeq (-)\sigma_h(p) \ \frac{\alpha_s C_F}{} \ \frac{dz}{} \ dk_T^2$$

$$\sigma_{V+h}(p) \simeq (-)\sigma_h(p)\ \frac{\alpha_s C_F}{\pi}\ \frac{dz}{1-z}\ \frac{dk_T^2}{k_T^2}$$

virtual cancels real emission divergence.

Total xsec:

$$\sigma_{g+h} + \sigma_{V+h} \simeq \frac{\alpha_s}{\pi} C_F \int \frac{dk_T^2}{k_T^2}\ \frac{dz}{1-z}\left(\sigma_h(zp) - \sigma(p)\right)$$

Now, the soft divergence in ISR: $z \to 1$, gluon momentum goes to 0.

$$\int \frac{dz}{1-z}\left(\sigma_h(zp) - \sigma(p)\right) \to \int \frac{dz}{1-z} \times 0 \sim \text{resolves soft divergence.}$$

But, the collinear divergence, $k_T^2 \to 0$: this is not resolved for any z.

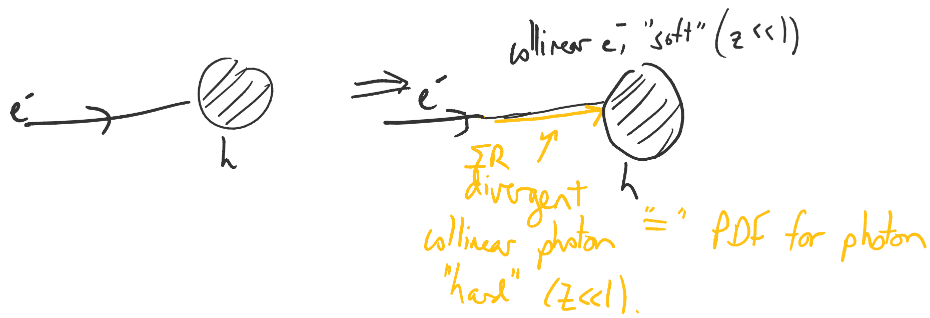
k_T^2 is the transverse momentum of the gluon relative to parent quark. Can be hard radiation in collinear region, with $z \to 0$.

$\theta \ll 1$.

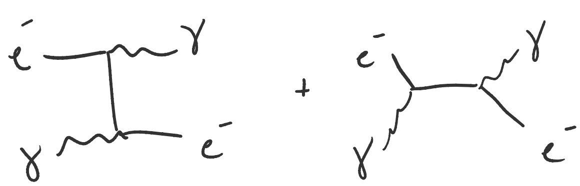
So, we will only focus on collinear divergence in ISR.

Start with QED:

Motivate the existence of γ-PDF arising from the regularization of collinear divergence in ISR.



Consider Compton scattering



High energy behavior:

COM

$k' = (\omega', \omega' s_\theta, 0, \omega' c_\theta)$ $\qquad$ $p\cdot k' = E\omega + \omega^2 c_\theta = \omega(E+\omega c_\theta)$

$k = (\omega, \omega\hat{z})$ $\quad \theta$ $\qquad$ $p \cdot k = E\omega + \omega^2$

$p = (E, -\omega\hat{z})$

$p' = (E', -\omega' s_\theta, 0, -\omega' c_\theta)$

$E'^2 = \omega'^2 + m^2$

$E^2 = \omega^2 + m^2$

$E' = \omega'\sqrt{1+\frac{m^2}{\omega'^2}} \simeq \omega'\left(1+\frac{m^2}{2\omega'^2}+\dots\right)$

drop $\qquad$ drop

$$\frac{1}{4}\sum_s |\mathcal{M}|^2 = 2e^4\left[\frac{p\cdot k'}{p\cdot k} + \frac{p\cdot k}{p\cdot k'} + 2m^2\left(\frac{1}{p\cdot k} - \frac{1}{p\cdot k'}\right) + m^4\left(\frac{1}{p\cdot k} - \frac{1}{p\cdot k'}\right)^2\right]$$

$E+\omega = \omega' + E'$

We want analyze singularities: $\theta \simeq \pi$, neglect $m_e \ll E$.

$E+\omega = 2\omega' + \frac{m^2}{2\omega'}$

For $m \ll E$, $m \ll \omega$

$\Rightarrow \omega = \omega'$

$E \sim \omega\left(1+\frac{m^2}{2\omega}\right)$

$\theta \simeq \pi \Rightarrow 0$

$$\frac{1}{4}\sum_s |\mathcal{M}|^2 = 2e^4\left[\frac{\omega(E+\omega c_\theta)}{\omega(E+\omega)} + \frac{\omega(E+\omega)}{\omega(E+\omega c_\theta)}\right]$$

$$= 2e^4 \frac{E+\omega}{E+\omega c_\theta}$$

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Problematic term in Compton Scattering

$$\frac{1}{4}\sum_s |\mathcal{M}|^2 = 2e^4\frac{E+\omega}{E+\omega c_\theta}$$

$$\frac{d\sigma}{d\cos\theta} = \frac{1}{2}\cdot\frac{1}{2E}\frac{1}{2\omega}\cdot\frac{\omega}{(2\pi)\,4\,(E+\omega)}\frac{2e^4(E+\omega)}{E+\omega\cos\theta}$$

$$\simeq \frac{2\pi\alpha^2}{2m^2+s(1+\cos\theta)}$$

General

$(\omega, \omega\hat{z})$ $\quad$ $(\omega', \omega' s_\theta, 0, \omega' c_\theta)$ $\quad \theta$

e $\quad (m, \vec{0})$

e $\quad (E', \vec{p}')$

$$\boxed{\frac{1}{\omega'} - \frac{1}{\omega} = \frac{1}{m}(1-c_\theta)}$$

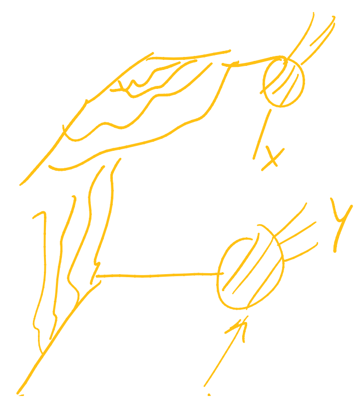
$$\omega' = \frac{\omega}{1+\frac{\omega}{m}(1-c_\theta)}$$

$\omega \gg m$, $\omega' \simeq \omega$ $\quad \times \quad$ not well-defined with $m \to 0$.

For HE scattering, $s \gg m^2$, denominator vanishes for $\theta \simeq \pi$.

Cut off by m_e: $$\int_{-1}^{1} d\cos\theta \frac{d\sigma}{dc_\theta} \simeq \frac{2\pi\alpha^2}{s}\int_{\boxed{-1+2m^2/s}}^{1} dc_\theta \frac{1}{1+c_\theta}$$

$$\sigma_{tot} = \frac{2\pi\alpha^2}{s}\log\left(\frac{s}{m^2}\right)$$



$$\sigma_{tot} = \frac{2\pi\alpha^2}{s}\log\left(\frac{s}{m^2}\right)$$

Collinear singularity:

$\alpha \ln\left(\frac{s}{m^2}\right)^n$

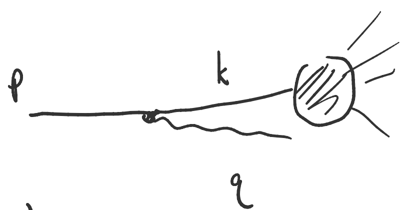
Important to note this log is not small.

Even worse for QCD: collinear gluon emission factor is $\alpha_s^2(Q^2)\log\left(\frac{Q^2}{\mu^2}\right)$

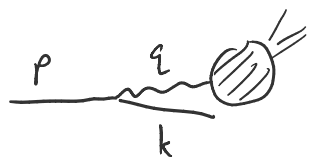
↳ factorization scale.

Tie into ISR diagrams:

General form is



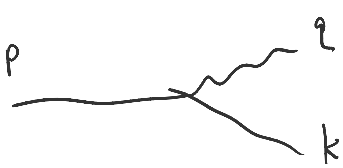
or



Technically, we will factorize on "almost" real polarization spinors or vectors for the intermediate e^- or photon.

Essentially, can factorize ISR splitting from hard process blob.

Kinematics of collinear splitting



$$p = (p, 0, 0, p)$$

$$q \simeq \left(zp, p_\perp, 0, zp - \frac{p_\perp^2}{2zp}\right)$$

$$k \simeq \left(\overset{E}{(1-z)p}, \overset{p_x}{-p_\perp}, \overset{p_y}{0}, \overset{p_z}{(1-z)p + \frac{p_\perp^2}{2zp}}\right)$$

By momentum conservation.

Keeps on-shell contraint true up to $O(p_\perp^4)$. (neglect m)

$$k^2 = \cancel{(1-z)^2 p^2} - \cancel{p_\perp^2} - \cancel{(1-z)^2 p^2} - 2(1-z)p \cdot \frac{p_\perp^2}{2zp} - \frac{p_\perp^4}{4zp}$$

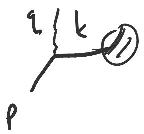
$$k \quad \cancel{(1-z)}\;\cancel{p} \quad p_\perp - \cancel{(1-z)^2}\, p^2 \underbrace{-2\,(1-z)p \cdot \frac{p_\perp^2}{2zp}} - \frac{p_\perp^4}{4zp}$$

$$-2p\,\frac{p_\perp^2}{2zp} + \cancel{\frac{2zp\, p_\perp^2}{2zp}}$$

photon real, electron virtual

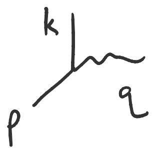
$$k^2 = \frac{-p_\perp^2}{z} + \mathcal{O}(p_\perp^4)$$

$$q^2 = \cancel{z^2p^2} - \cancel{p_\perp^2} - \cancel{z^2p^2} + \cancel{\frac{2zp\,p_\perp^2}{2zp}} - \frac{p_\perp^4}{4z^2p^2}$$



$$q^2 = 0 + \mathcal{O}(p_\perp^4)$$

By symmetry,



photon virtual, electron real

$$k^2 = 0$$

$$q^2 = \frac{-p_\perp^2}{1-z}$$

Break up numerator:

$$\not{k} = \sum_s u^s(k)\,\bar{u}^s(k)$$

$$\text{or} \quad \frac{-ig^{\mu\nu}}{q^2} \to \frac{+i}{q^2} \sum \epsilon^\mu_{T_i} \epsilon^{\nu *}_{T_i} \qquad \text{(real photon pols.)}$$

Just assign explicit ϵ_T + u pol. spinors + vectors.

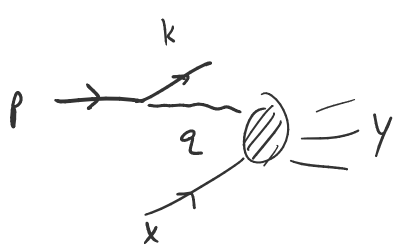
$$i\mathcal{M}\,(e^-_L \to e^-_L \gamma_R) = ie\,\frac{\sqrt{2(1-z)}}{z}\,p_\perp \qquad (17.90)$$

$$i\mathcal{M}\,(e^-_L \to e^-_L \gamma_L) = ie\,\frac{\sqrt{2(1-z)}}{z(1-z)}\,p_\perp \qquad (17.91)$$

Same for $L \leftrightarrow R$.

$$\Rightarrow \frac{1}{2}\sum_{\text{pols.}} |\mathcal{M}|^2 = \underbrace{\frac{2e^2 p_\perp^2}{z(1-z)}}\left[\frac{1+(1-z)^2}{z}\right]$$

this factor is a correction to the electron as a "parton".



$$\sigma = \frac{1}{2p\,2E_X\,(1+v_X)} \int \frac{d^3k}{(2\pi)^3}\cdot\frac{1}{2k^0}\cdot\int d\Pi_Y \left[\frac{1}{2}\sum|\mathcal{M}|^2\right]\left(\frac{1}{q^2}\right)^2 \cdot |\mathcal{M}_{\gamma X}|^2$$

splitting ME

$$\sigma = \frac{1}{2p\,2E_X(1+v_X)} \int \frac{d^3k}{(2\pi)^3\,2k^0} \cdots \Big[\underset{\uparrow}{\cdots} \Big] \cdots$$

splitting $|M|^2$ from above

Change variables for

$$d^3k = dk^3\, d^2k_\perp = p\, dz \cdot \pi\, dp_\perp^2$$

$$\sigma = \int \frac{dz\, dp_\perp^2}{16\pi^2(1-z)} \underbrace{\left[\frac{1}{2}\sum |M|^2\right]}_{\text{splitting}} \frac{z(1-z)^2}{p_\perp^4} \underbrace{\sigma(\gamma X \to Y)}_{\text{folded in } \frac{z}{1+v_X}\frac{1}{2zp}\frac{1}{2E_X}\int d\Pi_Y |M_{\gamma X}|^2}$$

$$\sigma = \int \frac{dz\, dp_\perp^2}{16\pi^2 \cancel{(1-z)}} \frac{2e^2 \cancel{p_\perp^2}}{\cancel{z(1-z)}} \cdot \frac{\cancel{z}\cancel{(1-z)}^2}{p_\perp^{\cancel{4}\,2}} \left[\frac{1+(1-z)^2}{z}\right]$$

$$\sigma = \int_0^1 dz \underbrace{\int \frac{dp_\perp^2}{p_\perp^2}}_{\substack{\text{singularity}\\ \text{cut off by } m}} \frac{\alpha}{2\pi} \left(\frac{1+(1-z)^2}{z}\right) \sigma(\gamma X \to Y)$$

$$\sigma(\underset{\substack{\uparrow\\ \text{now } \gamma \\ \text{on RHS}}}{e^-} X \to e^- Y) = \int_0^1 dz\ \frac{\alpha}{2\pi} \log\frac{s}{m^2} \left(\frac{1+(1-z)^2}{z}\right) \sigma(\gamma X \to Y)$$

In spirit,

$$\int_0^1 d\xi\ f_f\ \sigma(f(\xi p) \ldots)$$

$\int_0^1 dz$ is integral over momentum fraction.

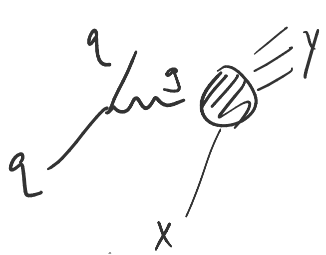
Motivates photon PDF:

$$f_\gamma(z) = \frac{\alpha}{2\pi} \log\left(\frac{s}{m^2}\right)\left[\frac{1+(1-z)^2}{z}\right]$$

$$= \int_0^1 dz\ f_\gamma(z)\ \sigma(\gamma(zp) X \to Y)$$

Called the Weizsäcker-Williams equivalent photon approx.

Exactly same logic



$f_g(z)$ expression.

q

x

Next time: modify the electron distribution to share phase space with f_γ.

Couple together in RG-style $\frac{d}{d\log Q^2}$ system of eqns.

Make jump to QCD & see DGLAP.