Lecture 11, January 6, 2021

Tuesday, January 5, 2021 2:09 PM

HW 4 now available,
discussion on Jan. 14.

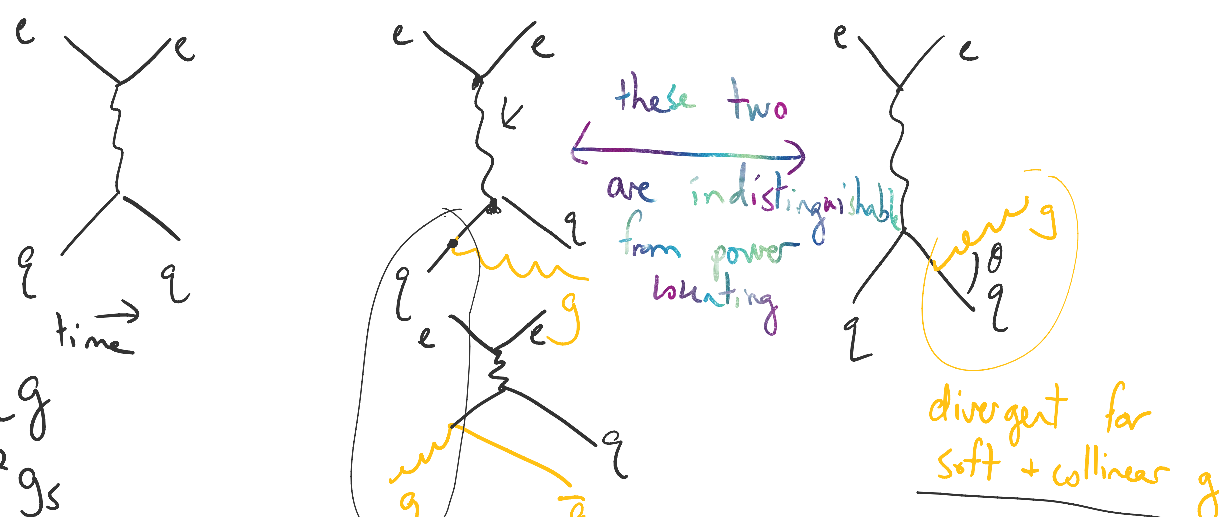
Today: Collider Kinematics
Dijet production

From previous lectures, recall we introduced parton model for
DIS (deep inelastic scattering).

$$\sigma\left(e^-(k)\,p(P) \to e^-(k') + X\right)$$
$$= \int_0^1 d\xi \sum_f f_f(\xi)\, \hat{\sigma}\left(e^-(k)\, q_f(\xi P) \to e^-(k') + q_f(p')\right)$$

$\hat{\sigma}$ = partonic cross section.
σ = total cross section.
f_f = PDFs.
ξ = momentum fraction

What happens at next order is α_s?
Necessarily involves corrections to hard process at α_s, but also new tree processes with ISR or FSR (initial state / final state radiation).



$eq \to eqg$
$\mathcal{M} \propto e^2 g_s$

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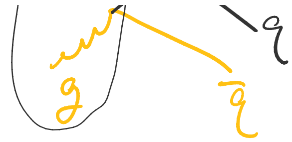


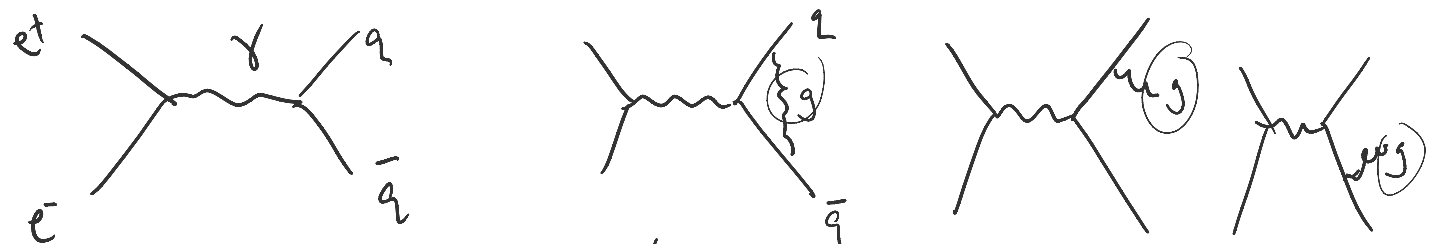
soft + collinear g

ISR divergences are folded into redefinition of PDFs. ⇒ Exactly analogous to counterterm in RG theory ⇒ Consequence is $\boxed{\text{PDFs evolution with } Q^2}$

ISR, either gluon radiation from initial parton or gluon splitting from gluon PDF

FSR, gluon radiation or splitting (need tree-level g in final state to get gluon splitting)

Have to divide phase space of FSR gluons to be hard + wide-angle.



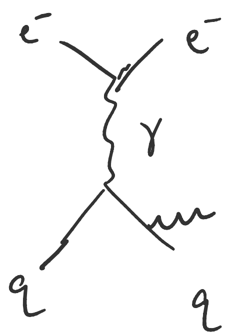
exclusive: $\sigma(e^+e^- \to 2\text{jets})$

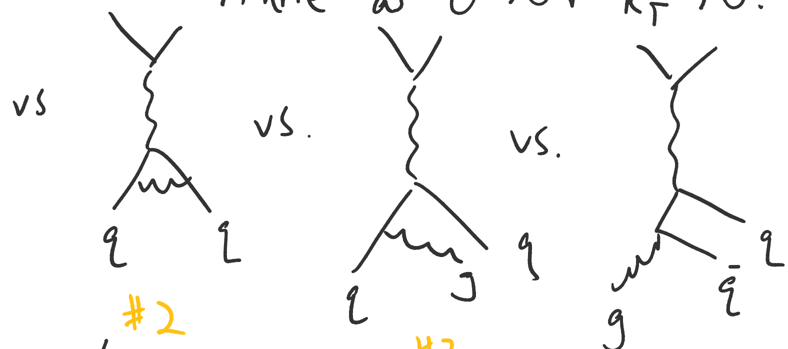
$\sigma(e^+e^- \to 3\text{jets})$

$\sigma(e^+e^- \to 4\text{jets} \ldots) \to$ as long as jet definition is IR safe

⇒ finite as $\theta \to 0$ & $k_T \to 0$.

Turn:





#1 #2 #3 #4

exclusive cross sections to resolve IR divergence.

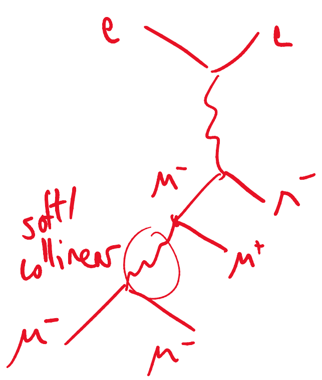
these are tied together by div. structure/evolution

but virtuality of γ is different in #3.

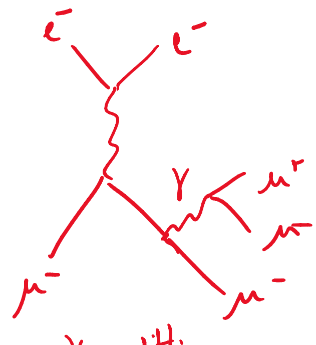
PDF in #1 & #3 have to evolve strictly to guarantee

PDF in #1 & #3 have to evolve strictly to guarantee the IR divergence cancels.

For QED + muons,
μ PDF + collinear/soft divs. for γ-radiation, and a γ-PDF.

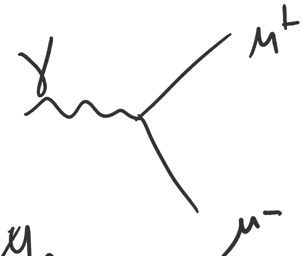


vs.

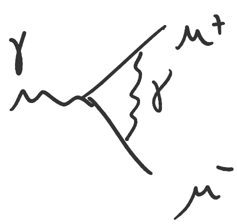


γ splitting

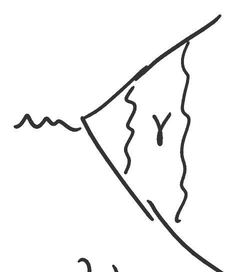
IR divergences are always present at any order in coupling
$\Rightarrow$ always have an all-orders resummation to remove.



$\mathcal{M} \sim e$
+



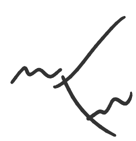
$\mathcal{M}_{\text{1-loop}} \sim e^3$



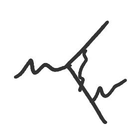
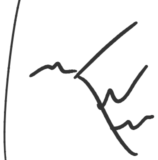
2-loop
$\mathcal{M}_{\text{2-loop}} \sim e^5$

Sudakov double log - finite resummation @ given $\mathcal{O}(\alpha^2)$.

$\left[\mathcal{M}_{\text{vtx, 1-loop}} (\mathcal{O}(\alpha^2)) \sim \text{IR-div.} \right.$
fixed by FSR @ tree



$\mathcal{M}_{\text{vtx, 2-loop}} (\mathcal{O}(\alpha^3)) \sim \text{IR-div.}$
fixed by FSR @ 1-loop

 + () not sure

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Break until 3:20pm

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Particle kinematics:

4-momentum (E, p_x, p_y, p_z)

Choose beam axis along $\hat{z}$ for collider,

$(E, p_z) \leftrightarrow (m_T, y)$

$E = m_T \cosh y, \quad p_z = m_T \sinh y.$

$m_T \equiv \sqrt{m^2 + p_x^2 + p_y^2}$ "transverse mass"

$y =$ "rapidity"

$$y = \frac{1}{2}\ln\left(\frac{E+p_z}{E-p_z}\right) = \ln\left(\frac{E+p_z}{m_T}\right) = \tanh^{-1}\left(\frac{p_z}{E}\right)$$

Under longitudinal boosts, differences in rapidity are invariant.

Related is pseudorapidity,

for $m \ll |\vec{p}| \Rightarrow$ $$y = \frac{1}{2}\ln\left(\frac{\cos^2(\theta/2) + m^2/4p^2 + \ldots}{\sin^2(\theta/2) + m^2/4p^2 + \ldots}\right)$$

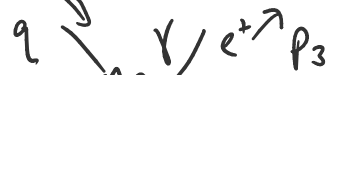
$$\simeq -\ln\tan\left(\frac{\theta}{2}\right) \equiv \eta$$

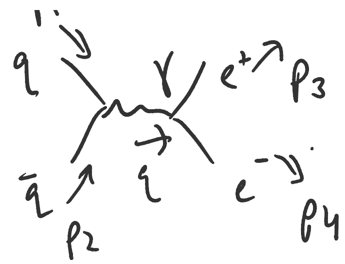
for $\cos\theta = \dfrac{p_z}{|\vec{p}|}$.

Consider pair production of l^+l^- from $q\bar{q}$:

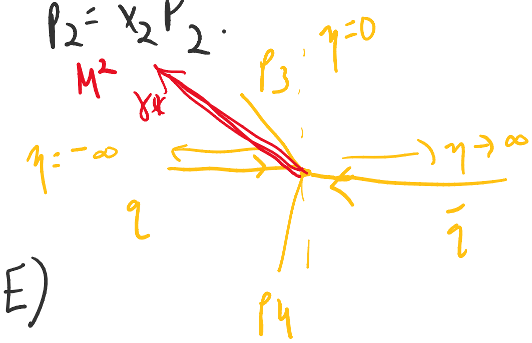
$$\sigma(q\bar{q} \to e^+e^-) = \frac{1}{3}Q_f^2 \frac{4\pi\alpha^2}{3\hat{s}}.$$

Given that we measure 4-momenta p_3 & p_4 of e^+ & e^-, we can know the virtuality of $q^2 = (p_3+p_4)^2 = M^2$





$$p_1 = x_1 P_1, \quad p_2 = x_2 P_2.$$



In COM frame of two protons:

$$P_1 = (E, 0, 0, E) \qquad P_2 = (E, 0, 0, -E)$$

$$q = \left((x_1+x_2)E,\ 0,\ 0,\ (x_1-x_2)E \right)$$

$$(p_3+p_4)^2 = q^2 = M^2 = x_1 x_2 4E^2 = x_1 x_2 s.$$

Since $x_1 \neq x_2$ in general, the lepton pair has a net rapidity in COM frame of pp collision:

$$E = M\cosh Y \Rightarrow \cosh Y = \frac{(x_1+x_2)E}{2E\sqrt{x_1x_2}} = \frac{1}{2}\left(\sqrt{\frac{x_1}{x_2}} + \sqrt{\frac{x_2}{x_1}}\right)$$

$$\Rightarrow \exp Y = \sqrt{\frac{x_1}{x_2}}.$$

We invert & determine x_1, x_2:

$$x_1 = \frac{M}{\sqrt{s}} e^{Y}, \quad x_2 = \frac{M}{\sqrt{s}} e^{-Y}.$$

Measured final state momenta $\Rightarrow$ determined x_1 & x_2.

Go back to total cross section:

$$\sigma\left(p(P_1) + p(P_2) \to e^+e^- + X\right)$$

$$= \int_0^1 dx_1 \int_0^1 dx_2 \sum_f f_q(x_1)\, f_{\bar{q}}(x_2)\, \hat{\sigma}\left(q_f(x_1 P)\, \bar{q}_f(x_2 P) \to e^+e^-\right)$$

We can change variables to M^2 & Y.

$$\frac{\partial^2 (M^2, Y)}{\partial x_1 \partial x_2} = \begin{vmatrix} x_2 s & x_1 s \\ 1 & -1 \end{vmatrix} = s = \frac{M^2}{x_1 x_2}$$

$$\frac{\partial(\ldots)}{\partial x_1\, \partial x_2} = \begin{vmatrix} \ldots & \ldots \\ \frac{1}{2x_1} & \frac{-1}{2x_2} \end{vmatrix} = s = \frac{\ldots}{x_1 x_2}$$

$$\frac{d^2\sigma}{d(M^2)\,dY}(pp \to e^+e^- + X) = \sum_f x_1 f_f(x_1)\, x_2 f_{\bar{f}}(x_2)\, \frac{1}{3} Q_f^2\, \frac{4\pi\alpha^2}{3M^4}$$

So, the doubly differential Drell-Yan process cross section gives granular information about PDFs.
(Note x_1 + x_2 are known!)

Next topic: dijet production.

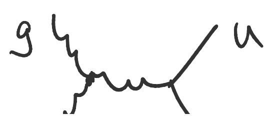
Instead of $e^+e^- \to q\bar{q}$, we want to discuss $\sigma(pp \to jj)$. As usual, wherever possible, we will adopt the equivalent QED calculation.

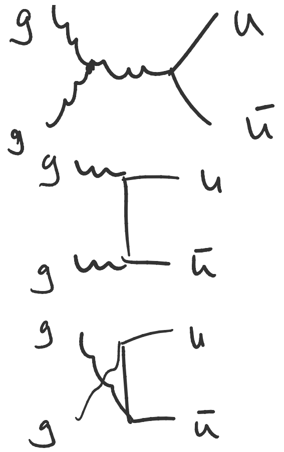
Complicated because p includes partons + jets form from quarks + gluons.

Simplify $N_F = 2$ QCD.

$u, d, g.$

QCD	QED	From P+S $\hat{\sigma}$
$u\bar{u} \to d\bar{d}$ $\;q\bar{q} \to q'\bar{q}'$ $(d\bar{d} \to u\bar{u})$	$e^+e^- \to \mu^+\mu^-$	(17.65) $\frac{d\sigma}{dt} = \frac{4\pi\alpha_s^2}{9\hat{s}^2}\left(\frac{\hat{t}^2 + \hat{u}^2}{\hat{s}^2}\right)$
$gg \to u\bar{u}$	$\gamma\gamma \to e^+e^-$	(17.76) $\frac{d\sigma}{dt} = \frac{\pi\alpha_s^2}{\ldots}$





(17.76)

$$\frac{d\sigma}{d\hat t} = \frac{\pi\alpha_s^2}{6\hat s^2}\left(\frac{\hat u}{\hat t}+\frac{\hat t}{\hat u}-\frac{9}{4}\left(\frac{\hat t^2+\hat u^2}{\hat s^2}\right)\right)$$

$gg \to gg$ —— (17.78)

$$\frac{d\sigma}{d\hat t} = \frac{9\pi\alpha_s^2}{2\hat s^2}\left(3-\frac{\hat t\hat u}{\hat s^2}-\frac{\hat s\hat u}{\hat t^2}-\frac{\hat s\hat t}{\hat u^2}\right)$$

$u\bar u \to u\bar u$ $e^+e^- \to e^+e^-$ (17.70)

$uu \to uu$ $e^-e^- \to e^-e^-$ (17.71)

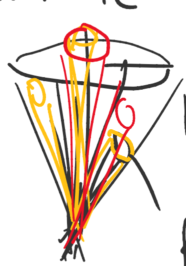
$qg \to qg$ $e^-\gamma \to e^-\gamma$ (17.77)

$\bar q g \to \bar q g$ $e^+\gamma \to e^+\gamma$ (17.77)

Jet definition $R = \sqrt{(\Delta\eta)^2+(\Delta\phi)^2}$

Choose seed partons : clustering $R < 0.8$
highest p_T.

IR-unsafe in jet substructure :



$R = 0.8$

$R = 0.2$

→ for a gien event, hard to apply a dimensionless cut.

Large grain analysis ⇒ computing

spectacular signal ⇒ should still

be spectacular at
low res.