Lecture 10, December 16, 2020

Wednesday, December 16, 2020 8:05 AM

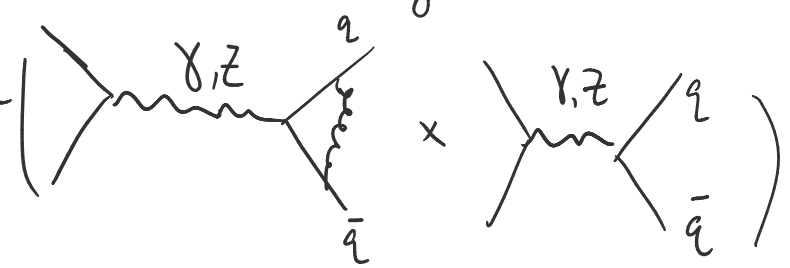
Last time: IR divergences & radiation of soft or collinear gauge bosons

Today: Concept of jets from quarks + gluons
Typical collider objects + detection efficiencies
Trigger, acceptance, & general collider terminology.

(Tomorrow: 9:30 am-11:30 am (c.t.),
will cover HW 3)

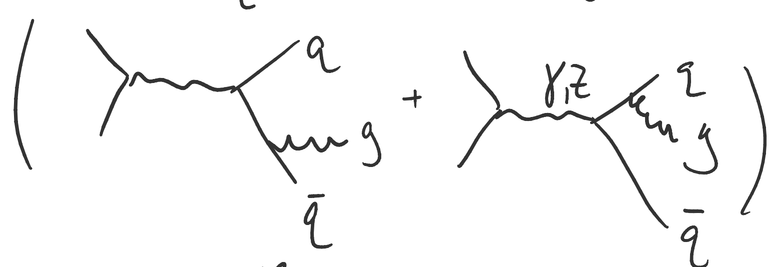
Analogy between hard scattering process of
$e^+e^- \to q\bar{q}$ (hadrons from e^+e^- collisions)
vs.
$e^+e^- \to \mu^+\mu^-$, known from QED.
Understand $\mathcal{O}(\alpha^2\alpha_s)$ result in $q\bar{q}$ production
through revisiting $\mathcal{O}(\alpha^3)$ result in $\mu^+\mu^-$.

$\frac{d\sigma}{d\Omega} \sim$



$\to \mathcal{O}(\alpha^2\alpha_s)$

$\frac{d\sigma}{d\Omega} \sim$



$\to \mathcal{O}(\alpha^2\alpha_s)$

IR divergence (familiar from vertex calc. in QED)
ties these two processes together.
Conclusion: sum of these is finite, each individual
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In other words, final states are always ambiguous by the presence of soft or collinear radiation.

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Scattering of electron with momentum p to momentum p' from photon with momentum transfer q:

$$\frac{d\sigma}{d\Omega} \simeq \left(\frac{d\sigma}{d\Omega}\right)_0 \left(1 - \underbrace{\frac{\alpha}{\pi} \log\left(\frac{-q^2}{m^2}\right) \log\left(\frac{-q^2}{\mu^2}\right)}_{\text{Sudakov double log}} + \mathcal{O}(\alpha^2) \right)$$

$q^2 < 0 \Rightarrow \mathcal{O}(\alpha)$ correction is negative + infinite

bremsstrahlung process

$$\frac{d\sigma}{d\Omega}\left(e(p) \to e(p') + \gamma\right) \simeq \left(\frac{d\sigma}{d\Omega}\right)_0 \left(\frac{\alpha}{\pi} \log\left(\frac{-q^2}{m^2}\right) \log\left(\frac{-q^2}{\mu^2}\right) + \mathcal{O}(\alpha^2) \right)$$

Sum of these processes is independent of μ^2 (fictitious photon mass) and finite!

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{measured}} \simeq \left(\frac{d\sigma}{d\Omega}\right)_0 \left(1 - \frac{\alpha}{\pi} f_{IR}(q^2) \log\left(\frac{-q^2 \text{ or } m^2}{E^2}\right) + \mathcal{O}(\alpha^2)\right)$$

would have zeros from pol. final state

enhancement factorizes from ang. momentum or spin sum.

where E = detection threshold for soft photons.

$\log\left(\frac{-q^2}{E^2}\right)$ or $\log\left(\frac{m^2}{E^2}\right)$ is a physical effect.

This enhanced IR divergence carries over from QED to QCD, where we just replace photons by gluons + include color factors.

So, we have to consider, for example

$\sigma(e^+e^- \to q\bar{q})$ and $\sigma(e^+e^- \to q\bar{q}g)$ together.

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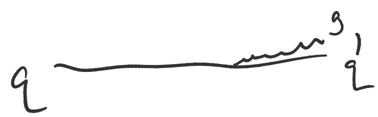
But realistically, sum is finite, actually only care about the case when the gluon in second process is below some energy threshold (or other cutoff) in order to include it in the first process.

Hence $\sigma(e^+e^- \to q\bar{q}) + \sigma(e^+e^- \to q\bar{q}g_{soft} \text{ or } q\bar{q}g_{collinear})$

and $\sigma(e^+e^- \to q\bar{q}g_{hard} \text{ or } q\bar{q}g_{wide\ angle})$

are distinct. Intuitively

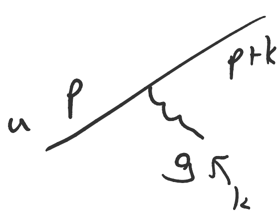
q_T (transverse momentum w.r.t. to quark momentum) small



, $\theta \ll 1$

Analyzing propagator $\Rightarrow$ motivate soft & collinear.

Preskel notes:



$$u(p)\not{\epsilon}\,\frac{\not{p}+\not{k}}{(p+k)^2-m^2} \sim u(p)\frac{p\cdot\epsilon}{m^2+2p\cdot k-m^2} \sim u(p)\frac{p\cdot\epsilon}{2p\cdot k}$$

$$p\cdot k \sim (E_q E_g - \vec{p}\cdot\vec{k})$$

$|\vec{k}| = E_g$ for $m_g = 0$

$$= (E_q E_g - |\vec{p}||\vec{k}|\cos\theta)$$

$$= E_g(E_q - |\vec{p}|\cos\theta)$$

soft div. for $E_g \to 0$

collinear div. for $m_f \to 0$]

$E_q = \sqrt{m_f^2 + |\vec{p}|^2}$

If W boson,

$E_W = \sqrt{m_W^2 + |\vec{k}|^2}$

$\Rightarrow E_q(E_W - |\vec{k}|\cos\theta)$

$\Rightarrow E_q(\sqrt{m_W^2 + |\vec{k}|^2} - |\vec{k}|\cos\theta)$

$\Rightarrow E_q|\vec{k}|$ collinear ...

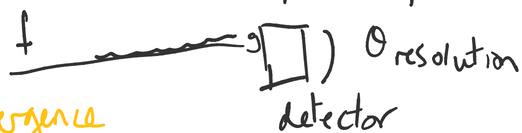
$\approx E_q E_g(1 - \cos\theta)$

$\to E_q\left(\sqrt{m_W^2+|k|^2} - |k|\cos\theta\right)$

$= E_q|\vec{k}|$
$\left(-\cos\theta + 1 + \frac{1}{2}\frac{m_W^2}{|\vec{k}|^2} + \dots\right)$
$\Downarrow$
Always get soft divergence before collinear divergence

$\approx E_q E_g(1-\cos\theta)$

collinear divergence:
resolved by finite size detector element,
$|\cos\theta| < 1-\epsilon$.



soft divergence: resolved by threshold energy on photons

Evaluating one process independently of the other ($q\bar{q}$ vs. $q\bar{q}g$) is divergent.
But ($q\bar{q}$ + $q\bar{q}g_{\text{resolution}}$) is finite, and divergent with threshold of $E_g \to 0$ & $\theta \to 0$.

Finite calculation for $q\bar{q}$ + soft/collinear,
$q\bar{q}$ + 1 hard g + soft/collinear, $q\bar{q}$ + $2g$ + soft/collinear, etc.
$\Rightarrow$ Perturbative calculations for additional gluon production (or photon production) according to multiplicity of hard gluons.
Implicitly, we have a detector threshold on energy resolution for soft gluons & angular threshold on collinear gluons.

Collider object that renders these issues moot is the <u>jet</u>: collection of hadrons that sums over all hadrons within some finite angle & some energy threshold for these hadrons.

Jet algorithms: Cambridge / Aachen, angular-ordered jet clustering

k_T

anti-k_T $\to$ soft-ordering algorithms.

First, high energy quarks + gluons radiate gluons and/or split into $q\bar{q}$ pairs.

Then, they hadronize when $\tau \sim \frac{1}{\Lambda_{QCD}}$.

Then, these hadrons (mesons & baryons) can further decay or be long-lived enough to interact with detector elements $\Rightarrow$ we will assume the detector reads momenta & energy very well.

Clustering algorithm: choose highest energy / hardest p_T hadron as clustering axis + keep adding in "neighboring" hadrons according to your jet radius choices.

"Neighboring" = close in angle (for C/A)
close in momentum fraction (for k_T, anti-k_T)

<u>IR safe</u> collider observable = jet.