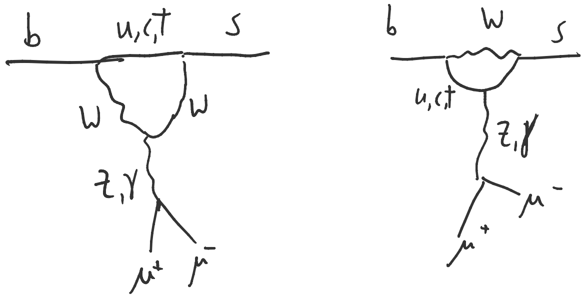
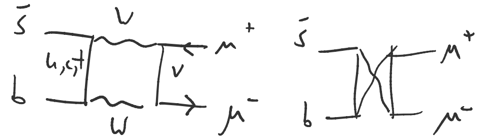
5.1. A. Diagram for $b \to s\,\mu^+\mu^-$

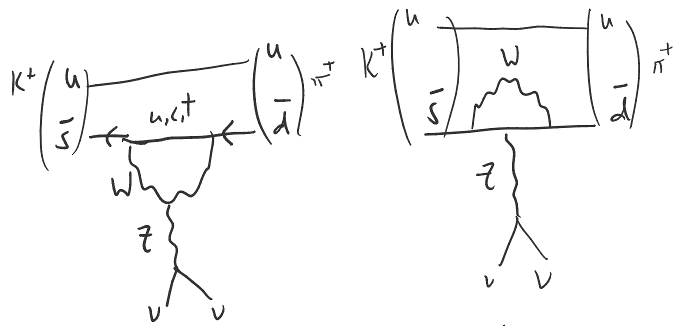


Note:
Also



B. $K^+ \to \pi^+ \nu\nu$

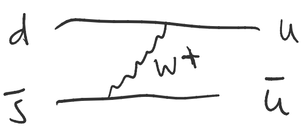
$K^+ = \begin{pmatrix} u \\ \bar{s} \end{pmatrix}$

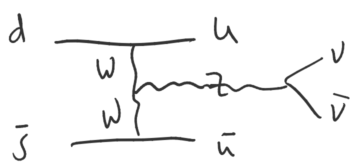


$K_L \to \pi^0 \nu\bar{\nu}$ $\qquad \pi^0 = \frac{(u\bar{u} + d\bar{d})}{\sqrt{2}}$

Change spectator u to d.

Note: weak coupling factorizes the interaction & picks out partonic content of meson.

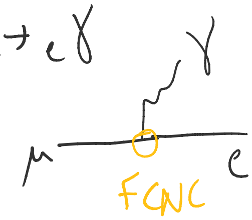




Naively better

$\frac{\Lambda^2_{QCD}}{m_W^2}$ factorization breaking?

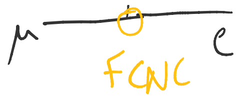
C.) $\mu \to e\gamma$

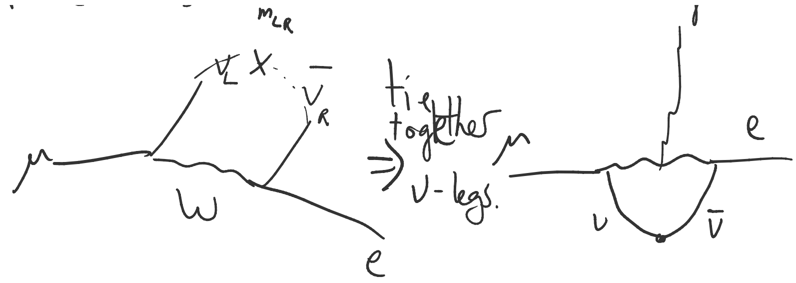


no neutrinos

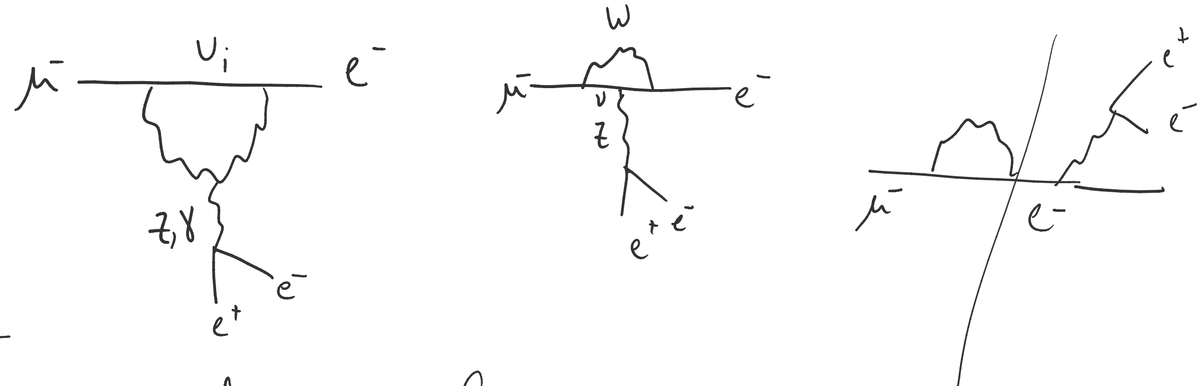
m_{LR}

$\nu_L \times \cdots \bar{\nu}$ ti...

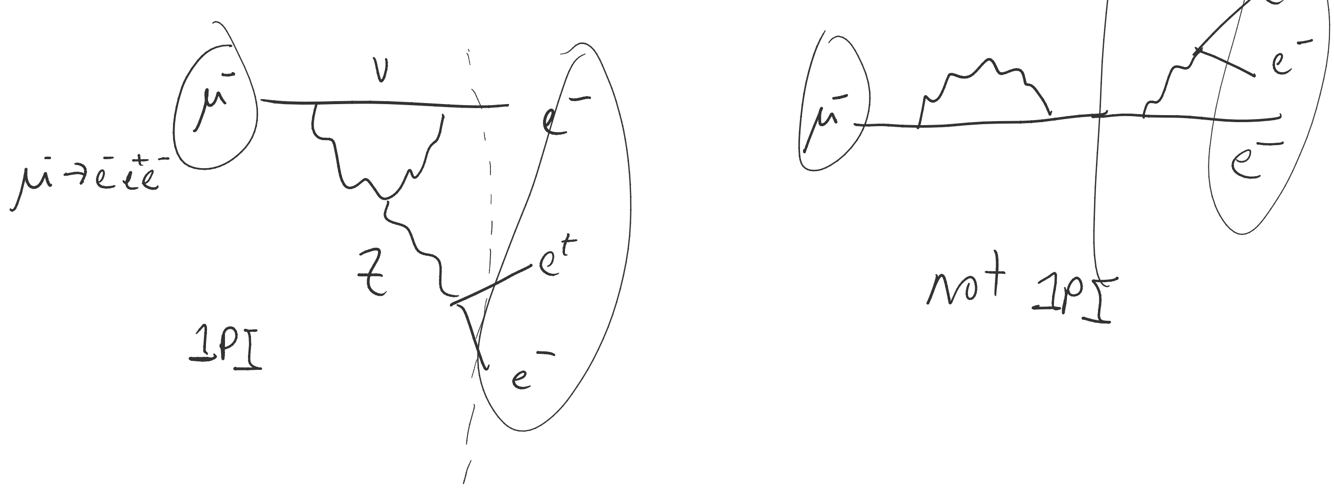




$\mu^- \to e^- e^+ e^-$



D. All penguin diagrams for FCNCs.



Estimate: $\mu \to e\gamma$

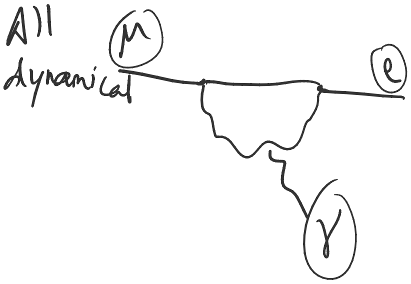
$$iM \sim \sum_i f\left(m_{\nu_i}^2, m_W^2, m_\mu^2, m_e^2\right) \cdot \underline{V_{\mu i} V_{ei}^*} \cdot \frac{1}{16\pi^2} \cdot g^2 \cdot e$$

$m_\mu^2, m_e^2)$ $\qquad$ $16\pi^2$

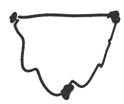
Main Trick: turn off external momenta + masses.

Then, can identify operator for loop fcn.

Can use NDA to get f(heavy scale masses) easily.



truncation of external legs.



focus only on algebraic structure of loop.

In general, loop fcn. is complicated.

For example, optical thm. guarantees loop fcn. has Im part when the internal legs are lighter than kinematic req. of ext. legs (go on-shell in loop).

Loop fcn. is (when appropriate scaling is removed) dimensionless

$\Rightarrow$ only fcns. of ratios of internal or ext masses + pure numbers (or $\frac{1}{\epsilon}$, if IR/UV divergence is uncancelled).

So for $\mu \to e\gamma$, decouples as $m_W^2 \to \infty$, so

must have $f(m_\nu^2, m_W^2, m_e^2, m_\mu^2) \sim f\left(\frac{m_\mu^2}{m_W^2}, \frac{m_\nu^2}{m_W^2}, \dots\right)$

For $m_\mu, m_e \to 0$, loop fcn.

reduces to $\frac{m_\nu^2}{m_W^2}$.

$$\Rightarrow i\mathcal{M} = V_{\mu i} V_{ei}^* \cdot \frac{m_\nu^2}{m_W^2} \cdot \frac{1}{16\pi^2} \cdot g^2 e$$

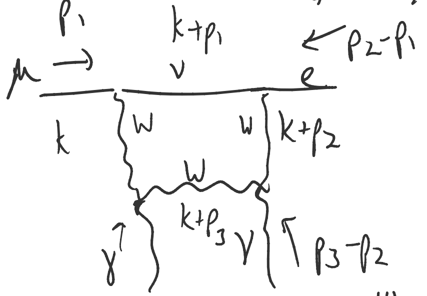
$$\Gamma \propto |\mathcal{M}|^2 = \left(\mathcal{O}(1)\right)^2 \left[\left(\frac{10^{-1}\,\text{eV}}{100\,\text{GeV}}\right)^2\right]^2 \cdot \left(\frac{1}{100}\right)^2 \cdot \left(\frac{1}{10}\right)^2$$

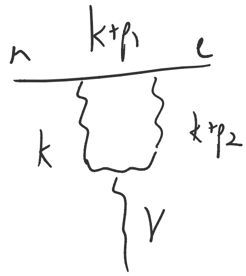
$$= \left[10^{-12}\right]^4 \cdot 10^{-8} \cdot 10^{-2}$$

$$\Gamma = 10^{-58}$$

$$\Gamma = 10^{-58}$$

$$Br\left(\frac{\Gamma(\mu\to e\gamma)}{\Gamma(\mu\to e\nu\nu)}\right) \sim 10^{-54}$$





$$f_4\left(p_1,p_2,p_3,\overset{\text{All}}{m_i}\right) \qquad f_3\left(p_1,p_2,\overset{\text{all}}{m_i}\right)$$

$$\lim f_4(p_3\to 0) = f_3 + \underline{\text{rational terms}}$$

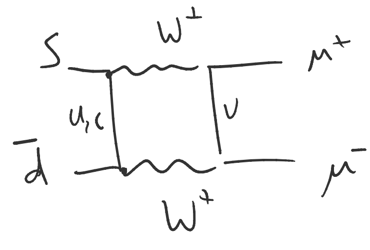
Discard all rational terms, $\quad$ (f_4 + f_3 not necessarily continuously connected)

then all loop fcns. are dimensionless fcns. of $\underset{0}{p_i}, \underbrace{m_i^{int}}, \underset{0}{m_i^{ext}}$

$\Rightarrow$ Can only have ratios of internal masses to match the dimensional scaling of amplitude.

5-2. GIM mechanism.

A) $\bar{K}_0 \to \mu^+\mu^-$



Finite cancellation extracts charm mass.

$$\mathcal{M}^{vtx} = \left(V_{us}V_{ud}^* f(m_u,m_W) + V_{cs}V_{cd}^* f(m_c,m_W)\right)\frac{1}{16\pi^2}\cdot g^4.$$

$$\begin{array}{c} \\ 2\times 2\ \text{CKM} = \end{array} \begin{array}{cc} d & s \\ c\theta_c & s\theta_c \end{array} \quad u$$

$$2\times 2 \; CKM = \begin{array}{c} \\ \end{array}\begin{pmatrix} c\theta_c & s\theta_c \\ -s\theta_c & c\theta_c \end{pmatrix} \begin{array}{c} u \\ c \end{array}$$

$$= \left(s\theta_c\, c\theta_c\, f(m_u, m_W) + c\theta_c(-s\theta_c)\, f(m_c, m_W) \right) \frac{1}{16\pi^2} g^4$$

$$\mathcal{M}^{vtx} = \left(s_{\theta_c} c_{\theta_c} \right) \left(f(m_u, m_W) - f(m_c, m_W) \right) \frac{1}{16\pi^2} g^4$$

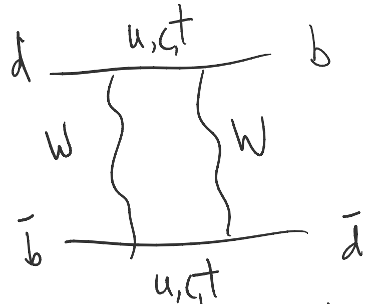
If $m_u = m_c$, vtx vanishes.

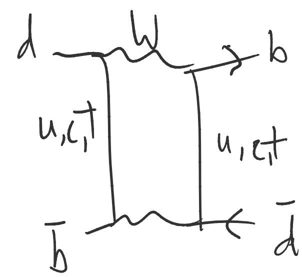
GIM mechanism.

Unitary nature of CKM implies that coherent sums of (same) loop function must scale as <u>differences</u> in quark masses.

IOW, degenerate quark masses restore flavor symmetry + eliminate FCNC effects.

b.) $B_0 - \bar{B}_0$





Essential diff. is what quark mass is used in loop fcn.

If $m_u = m_c = m_t \Rightarrow$ vanishes by CKM unitarity.