08.128.742 The Standard Model and Electroweak Theory Quantum Field Theory III

Felix Yu

Homework set 5

Due July 4, 2024 by start of discussion session
Please note how long it took you to solve each problem

5-1, 35 pts. The penguin diagram.

- A, 5 pts. Draw the 1-loop Feynman diagram for a transition of $b \to s\mu^+\mu^-$.
- B, 5 pts. Draw the 1-loop Feynman diagram for $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$.
- C, 5 pts. Draw the lowest order diagram for the $\mu \to e\gamma$ decay in the Standard Model, and also draw the $\mu \to 3e$ diagram.
- D, 10 pts. How are all of these diagrams (A)-(C) related? Explain the phenomenology of the penguin diagram, recognizing that the external states are different flavors.
- E, 10 pts. Estimate the branching fractions of (A)-(C) by naive dimensional analysis and taking into account the coupling dependencies. Compare with the SM experimental measurements or current exclusions (see the literature listed below). *Note:* The $B_s \to \mu^+\mu^-$ decay was one of the landmark measurements of the LHCb and CMS experiments at the LHC. Initial announcements were made in arXiv:1307.5024 (LHCb) and arXiv:1307.5025 (CMS), while a more recent analysis with more statistics can be found in arXiv:1703.05747. *Also note:* The $K \to \pi\nu\nu$ measurement is the primary motivation for the NA62 and KOTO experiments. There was initially some excitement about an excess in the KOTO dataset, but the reanalysis (after data unblinding) has increased the background estimate, which can be seen in arXiv:2012.07571. *Final note:* The $\mu \to e\gamma$ decay is the hallmark measurement of the MEG experiment, which concluded its physics run in 2013 and released a limit of 4.2×10^{-13} at 90% C.L. The analysis can be found in arXiv:1605.05081. MEG2 aims to improve the sensitivity by an order of magnitude.

5-2, 30 pts. Glashow-Iliopoulos-Maiani (GIM) mechanism.

- A, 15 pts. Draw the 1PI diagrams for the decay of $\bar{K}_0 \to \mu^+\mu^-$. If we restrict to a 2-generation set of up and down quarks, what is the prediction for the decay width for $\bar{K}_0 \to \mu^+\mu^-$? Relate this result to the structure of flavor symmetry in the 2-generation Standard Model. *Note:* This is the original calculation that led to the prediction of the charm quark.

B, 15 pts. Draw the 1PI diagrams for $B_0 \to \bar{B}_0$ oscillation. We will take the full SM with 3 generations of quarks. The easiest approach to evaluate the diagrams comes from setting the external momenta to zero and neglecting the light quark masses, including the bottom mass. Construct the expression for the mass difference between the B-meson mass eigenstates, in analogy to the $K_L - K_S$ mass difference.

5-3, 35 pts. Higgs boson production at the LHC. We will focus on understanding complications about extracting Higgs couplings at pp colliders.

A, 5 pts. Draw and list the five most dominant production modes at $\sqrt{s} = 13$ TeV and their cross sections. You can find the information from the LHC Higgs cross section working group.

B, 10 pts. The κ-framework is a straightforward modification of Higgs couplings to test experimentally for non-SM scattering rates of the 125 GeV Higgs boson. In this framework, all tree-level couplings of the Higgs boson are multiplied by their own κ_i, while the two leading loop-induced couplings of hgg and $h\gamma\gamma$ are also multiplied by κ_g and κ_γ, respectively. The SM case requires all $\kappa_i = 1$, while an experimental result favoring $\kappa_i \neq 1$ would indicate the presence of new physics. Write the Feynman rules for hgg, $h\gamma\gamma$, hW^+W^-, hZZ, and hbb in the κ framework.

C, 10 pts. Determine the cross section dependence on κ_i for $gg \to h \to \gamma\gamma$ in the κ-framework. Given a measurement of this event rate, can you individually determine the two corresponding κ parameters? What happens when you include a measured cross section for $gg \to h \to ZZ^*$? Does the situation improve when you add more decay modes or more production modes?

D, 10 pts. Identify the (non-unique) set of necessary assumptions in order to extract individual κ couplings from LHC data. *Hint:* Consider the role of the Higgs width. In general, the Higgs could decay to non-SM final states, which would be an additional contribution to the total Higgs width, Γ_{BSM}.

E, Bonus, 10 pts. At an e^+e^- collider, the additional kinematic certainty from the initial state allows an *inclusive* measurement of Higgs production in the $e^+e^- \to Zh$ channel. What observable gives this inclusive rate? How does having an inclusive rate for Higgs production now allow the κ-framework system of equations to close? *Hint:* Consider the system of equations from the inclusive rate measurement and the $e^+e^- \to Zh$, $h \to ZZ^*$ decay. This technique is known as the "recoil-mass" measurement and is a primary motivation for an e^+e^- Higgs factory.