08.128.742 The Standard Model and Electroweak Theory Quantum Field Theory III

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Homework set 3

Due June 6, 2024 at start of discussion session
Please note how long it took you to solve each problem

3-1, 50 pts. Goldstone Boson Equivalence. For this problem, you should read over Section 21.2 in Peskin and Schroeder and the relevant lecture notes discussing perturbative unitarity violation. The goal of this problem is to demonstrate the ease of approximating the longitudinal modes of massive EW gauge bosons by Goldstone bosons, which are explicitly realized by choosing the $R_\xi = 1$ gauge choice.

A, 20 pts. Calculate the $t \to bW^+$ decay width in the Standard Model in the unitarity gauge $R_\xi \to \infty$, setting the CKM factor $V_{tb} = 1$. The answer is in Eq. 21.76 of Peskin and Schroeder.

B, 20 pts. In any R_ξ gauge with finite ξ, we have explicit Goldstone bosons in the Higgs doublet field, which we write as

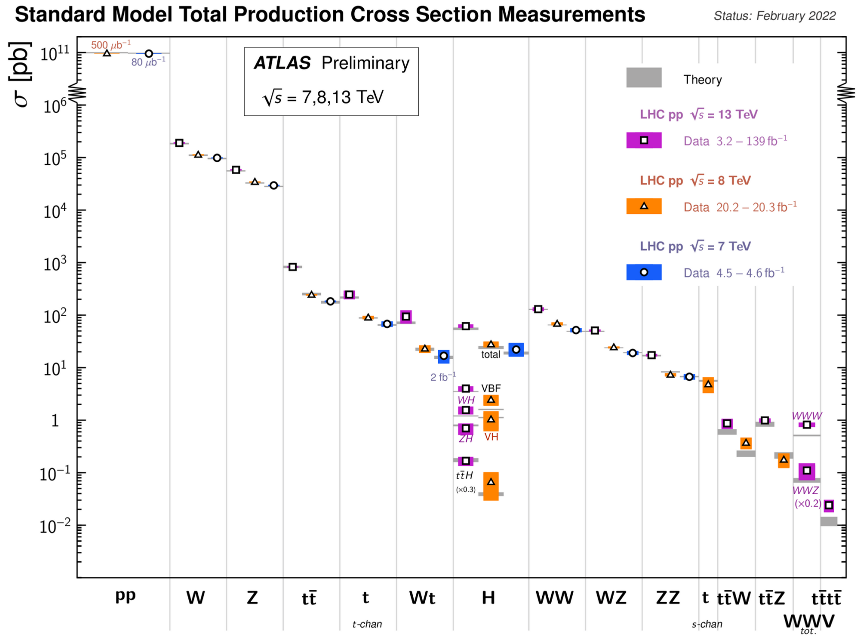
$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} -iG^+ \\ (v + h + iG^0) \end{pmatrix} , \tag{1}$$

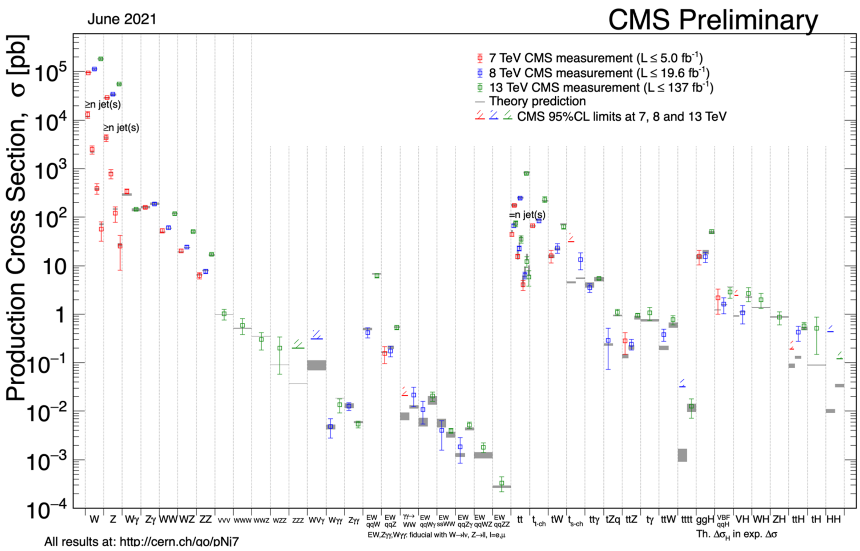
where G^+ is a complex scalar field and h is a CP-even real scalar field and G^0 is a CP-odd real scalar field. First, from the SM up-quark Yukawa interaction, write the specific Yukawa coupling between the charged Goldstone, top quark, and bottom quark, assuming the Yukawa matrix is already diagonalized. (The answer is written in Eq. 21.78.) Second, use this interaction to calculate the Feynman diagram and partial width for $t \to bG^+$ (the answer is given in Eq. 21.83 for $m_t \gg m_W$, but give the finite m_W answer assuming the G^+ mass is degenerate with m_W as for $R_\xi = 1$ gauge).

C, 10 pts. Show that the polarization vector sum in part (A) can be broken up into the purely transverse sum and the separate longitudinal sum, where the longitudinal sum is exactly the contribution of the Goldstone bosons calculated in part (B).

3-2, 50 pts. Standard Model cross sections and relative rates. To build intuition for collider process and the Standard Model, look at Figures and .

These figures show the cross section measurements for many SM processes at the LHC ranging from $\sqrt{s} = 7$, 8, and 13 TeV. Recall that the LHC is a proton-proton collider, and we will assume either a 4-flavor quark scheme where the proton parton





distribution function includes the gluon g, the quarks u, d, c, s, and their anti-quarks or a 5-flavor scheme where the PDF also includes the b and $\bar{b}$ quarks. Also, be aware that the "W" label usually sums over both W^+ and W^- bosons (except for WW, which is only the W^+W^- combination).

A, 10 pts. What is the rough scaling between single, double, and triple EW vector production cross sections? Why does the cross section scale this way?

B, 10 pts. What is the rough scaling between W production vs. Z production? How does this relate to WW, WZ, and ZZ production?

C, 10 pts. Draw the tree-level diagrams for $t\bar{t}$ production at the LHC, assuming that the proton PDFs use a 4-flavor scheme.

D, 15 pts. Draw the various tree-level 2-to-2 diagrams for single top production at the LHC, assuming that the proton PDFs use a 5-flavor scheme. Distinguish the diagrams according to whether the W boson is in the t-channel, s-channel, or produced in association with the top quark. (Representative diagrams can be found in arXiv:1902.07158.)

E, 5 pts. Which electroweak processes are expected to be asymmetric in electric charge and why? You can restrict your answer to considering single EW boson and double EW boson production processes only.