08.128.742 The Standard Model and Electroweak Theory Quantum Field Theory III

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Homework set 2

Due May 23, 2024 by start of discussion session
Please note how long it took you to solve each problem

2-1, 50 pts. Fermi theory and charged current interactions. In this problem, we will see a first example of an effective operator central to SM phenomenology, namely the charged current interaction mediated by the W boson and the corresponding effective operator.

A, 5 pts. Draw the Feynman diagram for tree-level muon decay and write the matrix element.

B, 15 pts. Expand the W propagator to leading powers of p^2/m_W^2, where p^2 is the momentum transfer carried by the W boson. Match the matrix element expression to the dimension-6 Fermi interaction and demonstrate that the Fermi effective operator gives the same leading order result at $1/m_W^2$ accuracy.

C, 15 pts. Generate the full Fermi theory by taking the $J^\mu_{W^+}$ and $J^\mu_{W^-}$ currents and multiplying them together (keep in mind the necessary CKM factors for quarks). Compare to the results from part (B) and explain why is this an equivalent formulation of the Fermi theory.

D, 15 pts. Write out the tree-level charged-current decays of a b quark using Fermi theory. If the b quark combines with a spectator u quark to form a B^+ meson, what final states do these charged-current decays predict and what is the rough relative rate between these different final states? (If you like, you can compare to the table of B^+ decay modes from the PDG.) Why is the charged current interaction dominant for b-quark decays?

2-2, 50 pts. Yukawa interactions and SM Effective Field Theory. One category of dimension-6 effective operators in the SMEFT (Standard Model Effective Field Theory) is the augmentation of any dimension-4 SM Lagrangian term by $H^\dagger H/\Lambda^2$, where H is the Higgs doublet and Λ is a scale of new physics.

A, 20 pts. Consider the SM down-quark Yukawa term and its $H^\dagger H/\Lambda^2$ dimension-6 EFT term,

$$\mathcal{L} = -y_d \overline{Q}_L H d_R - y'_d \frac{H^\dagger H}{\Lambda^2} \overline{Q}_L H d_R + \text{ h.c.}. \tag{1}$$

Using the Higgs vev insertion, what is the new down quark mass matrix that needs to be diagonalized?

B, 20 pts. Recognizing that the global symmetries are unchanged by the new dimension-6 EFT term, what are the new Yukawa couplings of the Higgs boson in the down quark mass basis? What requirements are needed on y_d and y'_d in order to have the Yukawa couplings be diagonal and CP conserving? (It is also interesting to think about the fine-tuning of fermion masses in the dim-6 SMEFT.)

C, 10 pts. SM trivia: What leading dim-6 operators correspond to the Higgs-digluon coupling and the Higgs-diphoton couplings? Why do these operators not correspond to a gluon mass or a photon mass?