08.128.742 The Standard Model and Electroweak Theory
Quantum Field Theory III

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Homework set 1

Due April 25, 2024 at the start of the discussion session.
Please note how long it took you to solve each problem.

1-1, 50 pts. Understanding the broken and unbroken phases of electroweak symmetry and the electroweak Lagrangian.

A, 20 pts. Using the mass basis for the electroweak gauge bosons, decompose the electroweak field strength kinetic terms into the corresponding kinetic field strengths for the $W^\pm$, Z, and γ gauge bosons, as well as the cubic and quartic interaction terms. (The answer is in equation 29.9 of Schwartz.) Namely, rewrite the unbroken phase electroweak field strength terms,

$$\mathcal{L} = -\frac{1}{4} W^i_{\mu\nu} W^{\mu\nu\ i} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} \ , \tag{1}$$

in the broken phase of electroweak symmetry.

B, 10 pts. From part (A), identify the interaction terms and write/look up the Feynman rules. (The answer is in the appendix of Cheng and Li.)

C, extra credit 10 pts. Derive the Feynman rules for part (B).

D, 20 pts. Perform the same procedure as part (A) on the Higgs kinetic term,

$$\mathcal{L} = |D_\mu H|^2 \ , \tag{2}$$

by expanding the Higgs doublet field around the Higgs vev, $H = \frac{1}{\sqrt{2}} (0 \quad (v+h))^T$, where h is the Higgs scalar excitation. (The answer is given in equation 29.14 of Schwartz.) Identify the interactions between the Higgs scalar and the electroweak gauge bosons and write their Feynman rules.

E, extra credit, 5 pts. How would the answer to part (A) change if we introduced a W' gauge boson that had a mixing angle with the SM W boson? For concreteness, you can consider adding an explicit gauged $SU(2)_R$ group.

1-2, 50 pts. Higgs physics and non-decoupling.

A, 35 pts. Calculate the $h \to gg$ decay width at one loop mediated by the top quark. (The answer is in equation 2.58 of arXiv/0503172 by A. Djouadi. You can check that your one-loop matrix elements are equivalent to the expression in equation 24 of arXiv:1205.4244 by Kumar, Vega Morales and FY.)

B, 10 pts. Demonstrate that an infinitely heavy top quark still gives a non-vanishing contribution to the $h \to gg$ decay width. (This equally implies that an infinitely heavy top quark would give a non-vanishing contribution to $gg \to h$ production.) This is known as *non-decoupling.*

C, 5 pts. In the Standard Model, the number of chiral generations is not known a priori. How would you probe the existence of a fourth (or more) generations of chiral fermions? Is it allowed or excluded by the current ATLAS and CMS collider results?