Special Lecture Topic: CP Violation Discovery and Neutral Kaon Oscillation

Quantum Field Theory II.
Felix Yu
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Themes to cover:
- 1-loop mixing of $K^0 - \bar{K}^0$
- Indirect CP violation
- Direct CP violation
- Briefly, time evolution + box diagram expression & hadronic matrix element

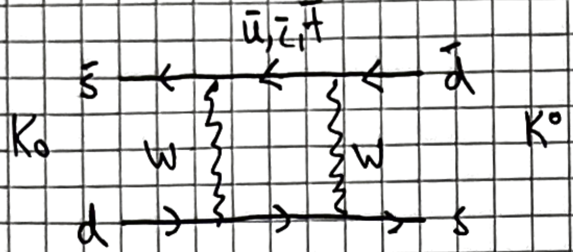
We have seen that one of the features of one-loop corrections / calculations in the SM is the possibility that particles mix at 1-loop order. Specifically, we noted that precision EW physics observables on the Z-pole required making 1-loop accurate predictions involving $Z-\gamma$ mixing. Other scenarios include the possibility of mixing the Higgs boson with scalar mesons of QCD, if the Higgs mass was kept as a free parameter.

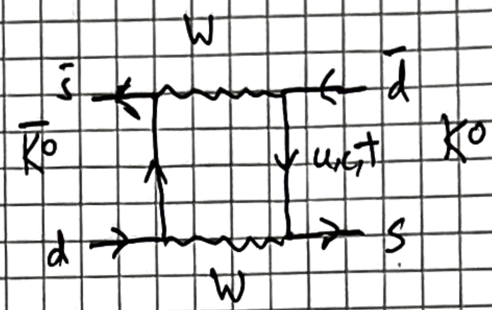
The specific system we want to consider in this lecture is the $K^0 - \bar{K}^0$ system. We can readily draw the loop diagrams that mix these states.

Note $K^0 = \begin{pmatrix} \bar{s} \\ d \end{pmatrix}$ and $\bar{K}^0 = \begin{pmatrix} s \\ \bar{d} \end{pmatrix}$

(while $D^0 = \begin{pmatrix} c \\ \bar{u} \end{pmatrix}$, $\bar{D}^0 = \begin{pmatrix} \bar{c} \\ u \end{pmatrix}$, $B_d^0 = \begin{pmatrix} \bar{b} \\ d \end{pmatrix}$, $\bar{B}_d^0 = \begin{pmatrix} b \\ \bar{d} \end{pmatrix}$).

Two box diagrams: $\Delta S = 2$





We note that the box diagrams can also be easily modified to reflect 1-loop self-energy diagrams that correct $K^0 - K^0$ and $\bar{K^0} - \bar{K^0}$ wavefcns.

In lieu of solving the loop diagrams explicitly, we can schematically solve the mixed system.

Following Pich, hep-ph/ 9312297.

$$\mathcal{M} = \begin{pmatrix} M & M_{12} \\ M_{12}^* & M \end{pmatrix} - \frac{i}{2} \begin{pmatrix} \Gamma & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma \end{pmatrix}$$

Diagonal elements are real, M & $\Gamma \in \mathbb{R}$.
If CP were conserved, M_{12} + Γ_{12} would also be real.

Can diagonalize states

$$|K_{S,L}\rangle = \frac{1}{\sqrt{|p|^2 + |q|^2}} \left[p\,|K^0\rangle \mp q\,|\bar{K^0}\rangle \right]$$

with
$$\frac{q}{p} \equiv \frac{1 - \bar{\epsilon}}{1 + \bar{\epsilon}} = \left(\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}} \right)^{1/2}$$

For M_{12} and Γ_{12}, then $\frac{q}{p} = 1$ and

$|K_{S,L}\rangle$ would correspond as CP-even K_1 + CP-odd K_2 states. $|K_{1,2}\rangle = \left(\frac{|K^0\rangle \mp |\bar{K^0}\rangle}{\sqrt{2}} \right)$.

Convention: $CP\,|K^0\rangle = -|\bar{K}^0\rangle$.

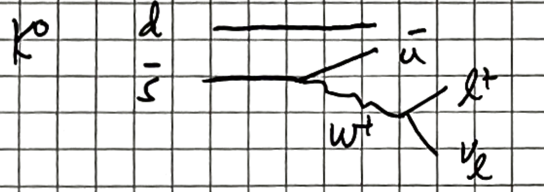
Since $K^0-\bar{K}^0$ violates CP, the mass eigenstates are no longer orthogonal,

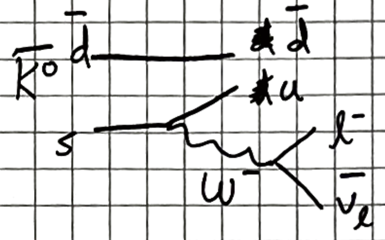
$$\langle K_L | K_S \rangle = \frac{|p|^2-|q|^2}{|p|^2+|q|^2} \approx 2\,\mathrm{Re}(\bar{\epsilon}).$$

We can thus probe <u>indirect CP violation</u> by noting that the semileptonic decay,

$$K^0 \to \pi^- \ell^+ \nu_\ell \quad , \quad \bar{K}^0 \to \pi^+ \ell^- \bar{\nu}_\ell$$

are distinct in the lepton charge depending on the parent K^0 / $\bar{K}^0$.





So, if we produce a final state of ℓ^+, we know it tracks the K^0 component of the mixed state. We have $|A(\bar{K}^0 \to \pi^+ \ell^- \bar{\nu}_\ell)| = |A(K^0 \to \pi^- \ell^+ \nu_\ell)|$ in the SM, so we can construct an observable for <u>indirect</u> <u>CPV</u>

$$\delta \equiv \frac{\Gamma(K_L \to \pi^- \ell^+ \nu_\ell) - \Gamma(K_L \to \pi^+ \ell^- \bar{\nu}_\ell)}{\Gamma(K_L \to \pi^- \ell^+ \nu_\ell) + \Gamma(K_L \to \pi^+ \ell^- \bar{\nu}_\ell)}$$

$$= \frac{|p|^2-|q|^2}{|p|^2+|q|^2}$$

$$= \frac{2\,\mathrm{Re}(\bar{\epsilon})}{1+|\bar{\epsilon}|^2}$$

Exp. gives $\delta = (3.27 \pm 0.12)\cdot 10^{-3}$, $\mathrm{Re}(\bar{\epsilon}) = (1.63 \pm 0.06)\cdot 10^{-3}$

Note the mass splitting between K_L & K_S is about $\Delta m \sim 3.5 \cdot 10^{-15}$ GeV.

This is called <u>indirect</u> CPV, since it reflects CPV induced by mixing CP-conjugate states.

In contrast, we can test for <u>direct</u> CPV. In this case, we hope to measure the decay rates of two CP-conjugate rates to be unequal.

So, concretely, we want $\Gamma(P \to f) \neq \Gamma(\bar{P} \to \bar{f})$.

Since at least two interfering amplitudes are needed to generate a CPV effect, we can write

$$A(P \to f) = M_1 e^{i\phi_1} e^{i\alpha_1} + M_2 e^{i\phi_2} e^{i\alpha_2}$$

$$A(\bar{P} \to \bar{f}) = M_1 e^{-i\phi_1} e^{i\alpha_1} + M_2 e^{-i\phi_2} e^{i\alpha_2}$$

Here, we have introduced ϕ_1 & ϕ_2 as <u>weak</u> phases, while α_1 & α_2 are <u>strong</u> phases.

"Weak" and "strong" are <u>not</u> related to weak & strong gauge groups.

Instead, "weak" phases are phases arising from Lagrangian params., i.e. they show up in Feynman rules, and they are the <u>interesting</u> physics for CPV.

How many CPV phases do we have in SM?

δ_{CP} for CKM. $\bar{\theta}$ from QCD + EW

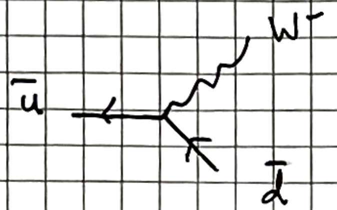
δ_{CP}, δ_{m_1}, δ_{m_2} for PMNS.

How does δ_{CP} in CKM show up in Feynman rules?

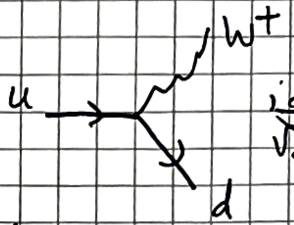
One param. of CKM is $R_{12} \cdot R_{13} \exp(i\delta) \cdot R_{23}$.

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix}$$

Now, suppose we consider Feynman rules using CKM elements.



$\frac{ig}{\sqrt{2}} V_{ud}$



$\frac{ig}{\sqrt{2}} V_{ud}^*$

The conjugation of the phase in the Feynman rule is directly connected to the term in the Lagrangian and its Hermitian conjugate. So the weak phase changes sign in the Feynman rule.

Strong phases, on the other hand, arise from particles going on-shell in loop amplitudes, and does not change sign.

We can calculate $\Gamma(P \to f)$ and $\bar{\Gamma}(\bar{P} \to \bar{f})$.

$$\begin{aligned} \Gamma(P \to f) &= |A(P \to f)|^2 \\ &= |M_1|^2 + |M_2|^2 + M_1 M_2 \exp(i(\phi_1 - \phi_2)) \exp(i(\alpha_1 - \alpha_2)) \\ &\quad + M_1 M_2 \exp(i(\phi_2 - \phi_1)) \exp(i(\alpha_2 - \alpha_1)) \end{aligned}$$

$$\Gamma(\bar{P} \to \bar{f}) = |A(\bar{P} \to \bar{f})|^2$$
$$= |M_1|^2 + |M_2|^2 + M_1 M_2 \exp(-i(\phi_1 - \phi_2)) \exp(i(\alpha_1 - \alpha_2))$$
$$+ M_1 M_2 \exp(-i(\phi_2 - \phi_1)) \exp(i(\alpha_2 - \alpha_1))$$

So, we can construct

$$\frac{\Gamma(P \to f) - \Gamma(\bar{P} \to \bar{f})}{\Gamma(P \to f) + \Gamma(\bar{P} \to \bar{f})}$$

$$= \frac{M_1 M_2 \left([\exp(i(\phi_1 - \phi_2)) - \exp(-i(\phi_1 - \phi_2))] \exp(i(\alpha_1 - \alpha_2)) + [\exp(i(\phi_2 - \phi_1)) - \exp(-i(\phi_2 - \phi_1))] \exp(i(\alpha_2 - \alpha_1)) \right)}{2|M_1|^2 + 2|M_2|^2 + M_1 M_2 [\exp(i(\phi_1 - \phi_2)) \exp(i(\alpha_1 - \alpha_2)) + \exp(i(\phi_2 - \phi_1)) \exp(i(\alpha_2 - \alpha_1)) + \exp(-i(\phi_1 - \phi_2)) \exp(i(\alpha_1 - \alpha_2)) + \exp(-i(\phi_2 - \phi_1)) \exp(i(\alpha_2 - \alpha_1))]}$$

$\exp(i\beta) - \exp(-i\beta)$
$= 2i \sin\beta$

$$= \frac{M_1 M_2 \left(\exp(i(\alpha_1 - \alpha_2))\, 2i \sin(\phi_1 - \phi_2) + \exp(i(\alpha_2 - \alpha_1))\, 2i \sin(\phi_2 - \phi_1) \right)}{2|M_1|^2 + 2|M_2|^2 + M_1 M_2 [\exp(i(\alpha_1 - \alpha_2))\, 2\cos(\phi_1 - \phi_2) + \exp(i(\alpha_2 - \alpha_1))\, 2\cos(\phi_2 - \phi_1)]}$$

$$= \frac{M_1 M_2 \; 2i \sin(\phi_1 - \phi_2)(2i) \sin(\alpha_1 - \alpha_2)}{2|M_1|^2 + 2|M_2|^2 + M_1 M_2 \; 4\cos(\phi_1 - \phi_2)\cos(\alpha_1 - \alpha_2)}$$

$$= \frac{-2 M_1 M_2 \sin(\phi_1 - \phi_2) \sin(\alpha_1 - \alpha_2)}{|M_1|^2 + |M_2|^2 + 2 M_1 M_2 \cos(\phi_1 - \phi_2)\cos(\alpha_1 - \alpha_2)}$$

We see that <u>direct</u> CPV requires at least two interfering amplitudes, two diff. weak phases + two diff. strong phases.

We note that the physical observable uses K_L + K_S states, + this requires taking into account direct + indirect CPV simultaneously.

Finally, we note the time evolution.

$$A(K^0 \to f) \sim \left\{ A(K_S \to f) e^{-iM_S t} e^{-\Gamma_S t/2} + A(K_L \to f) e^{-iM_L t} e^{-\Gamma_L t/2} \right.$$
$$A(K_S \to f) e^{-iM_S t} e^{-i\Gamma_S t/2} - A(K_L \to f) e^{-iM_L t} e^{-\Gamma_L t/2}$$

Use $\eta_f \equiv \dfrac{A(K_L \to f)}{A(K_S \to f)} \equiv |\eta_f| e^{i\phi_f}$

$$\Gamma(K^0 \to f) \sim e^{-\Gamma_S t} + |\eta_f|^2 e^{-\Gamma_L t} + 2|\eta_f| \cos(\phi_f - \Delta M t) e^{-(\Gamma_L + \Gamma_S)t/2}$$

$$\Gamma(\bar{K}^0 \to \bar{f}) \sim e^{-\Gamma_S t} + |\eta_f|^2 e^{-\Gamma_L t} - 2|\eta_f| \cos(\phi_f - \Delta M t) e^{-(\Gamma_L + \Gamma_S)t/2}$$

Returning to the box diagrams,

$$M_{12} = \frac{G_F^2 M_W^2}{16\pi^2} \left\{ \lambda_c^2 \eta_1 S(r_c) + \lambda_t^2 \eta_2 S(r_t) + 2\lambda_c \lambda_t \eta_3 S(r_c, r_t) \right\}$$
$$\cdot \frac{1}{2M_K} \alpha_s(\mu^2)^{-2/9} \cdot \langle \bar{K}^0 | (\bar{s} \gamma^\mu (1-\gamma_5) d)$$
$$(\bar{s} \gamma_\mu (1-\gamma_5) d) | K^0 \rangle$$

$$\lambda_i \equiv V_{is} V_{id}^*$$

+ $S(r_i)$, $S(r_i, r_j)$ are fcns. of $r_i \equiv \left(\dfrac{m_i}{m_W}\right)^2$.

~~Ea~~ η_i reflect short-distance QCD, $\eta_i = 1$ in absence of QCD effects. $\eta_1 \simeq 0.85$, $\eta_2 \simeq 0.57$, $\eta_3 \simeq 0.36$

Also need hadronic matrix element.

$$\alpha_s(\mu^2)^{-2/9} \langle \bar{K}^0 | (\bar{s} \gamma^\mu (1-\gamma_5) d)(\bar{s} \gamma_\mu (1-\gamma_5) d) | K^0 \rangle$$
$$\equiv 2\left(1 + \tfrac{1}{3}\right)(\sqrt{2} f_K M_K)^2 B_K,$$

where $B_K = 1$ is pure factorization of two currents.

Ranges $0.33 \sim 0.8$ using χPT to lattice.