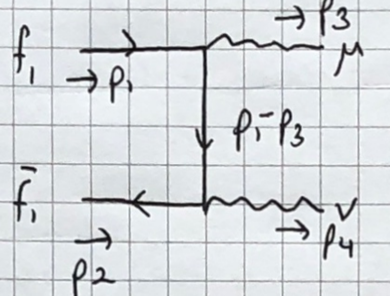
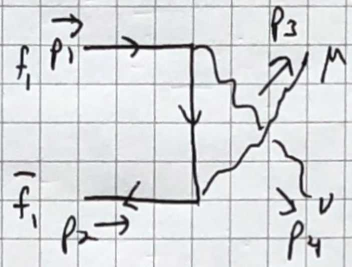
QFT 3. The Standard Model + EW Theory.
Lecture 5.

$f\bar{f} \to VV$ scattering

18.11.20

Calculate $\bar{f}f \to VV$ with approximate longitudinal polarization





$$i\mathcal{M}_1 = \bar{v}(p_2)\, i(g_V\gamma^\nu + g_A\gamma^\nu\gamma^5)\, \frac{i(\not{p}_1 - \not{p}_3 + m_2)}{t - m_2^2}\, i(g_V\gamma^\mu + g_A\gamma^\mu\gamma^5)\, u(p_1)\, \epsilon^*_\mu(p_3)\, \epsilon^*_\nu(p_4)$$

$t = (p_1 - p_3)^2$

$u = (p_1 - p_4)^2$

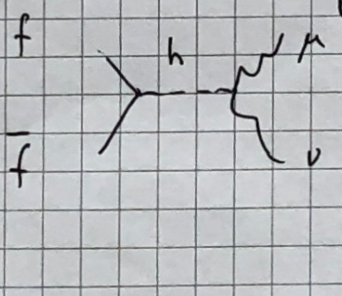
$$i\mathcal{M}_2 = \bar{v}(p_2)\, i(g_V\gamma^\mu + g_A\gamma^\mu\gamma^5)\, \frac{i(\not{p}_1 - \not{p}_4 + m_2)}{u - m_2^2}\, i(g_V\gamma^\nu + g_A\gamma^\nu\gamma^5)\, u(p_1)$$

$$\epsilon^*_\mu(p_3)\, \epsilon^*_\nu(p_4)$$

$$\epsilon^*_\mu(p_3) \approx \frac{p_3^\mu}{m_Z} - \frac{2m_Z}{s} p_4^\mu$$

$$\epsilon^*_\nu(p_4) \approx \frac{p_4^\mu}{m_Z} - \frac{2m_Z}{s} p_3^\mu$$

$$i\mathcal{M}_{1+2} = -i\bar{v}(p_2)\Big[\frac{1}{t - m_2}\Big(\frac{1}{m_W}\big(\not{p}_4 - \frac{2m_W^2}{s}\not{p}_3\big)(g_V + g_A\gamma^5)$$
$$(\not{p}_1 - \not{p}_3 + m)$$
$$\Big(\frac{1}{m_W}\big(\not{p}_3 - \frac{2m_W^2}{s}\not{p}_4\big)(g_V + g_A\gamma^5)\Big)$$
$$+ \frac{1}{u - m_2}\Big(\frac{1}{m_W}\big(\not{p}_3 - \frac{2m_W^2}{s}\not{p}_4\big)(g_V + g_A\gamma^5)$$
$$(\not{p}_1 - \not{p}_4 + m)$$
$$\Big(\frac{1}{m_W}\Big)\big(\not{p}_4 - \frac{2m_W^2}{s}\not{p}_3\big)(g_V + g_A\gamma^5)\Big)\Big]$$



$u(p_1)$

$$i\mathcal{M}_h = \bar{v}(p_2) \cdot -\frac{i m_f}{v}\, u(p_1) \cdot \frac{2i m_Z^2}{v} g^{\mu\nu}\, \epsilon^*_\mu(p_3)\epsilon^*_\nu(p_4) \cdot \frac{i}{s - m_h^2}$$

$$i\mathcal{M}_h = i\bar{v}(p_2)\,u(p_1)\cdot \frac{2m_f m_Z^2}{v^2}\,\frac{1}{s-m_h^2}\left(\frac{1}{m_Z^2}\right)\left(p_3^\mu - \frac{2m_Z^2}{s}p_4^\mu\right)\left(p_{4\mu} - \frac{2m_Z^2}{s}p_{3\mu}\right)$$

$$= i\bar{v}(p_2)\,u(p_1)\cdot \frac{2m_f}{v^2}\,\frac{1}{s-m_h^2}\left(p_3\cdot p_4 - \frac{2m_Z^2}{s}\cdot 2m_Z^2 + \frac{4m_Z^4}{s^2}\,p_3\cdot p_4\right)$$

$$p_3\cdot p_4 = \frac{1}{2}\left[(p_3+p_4)^2 - p_3^2 - p_4^2\right] = \frac{1}{2}(s - 2m_Z^2)$$

$$= i\bar{v}(p_2)\,u(p_1)\left[\frac{2m_f}{v^2}\,\frac{1}{s-m_h^2}\left(-\frac{4m_Z^4}{s} + \frac{1}{2}(s-2m_Z^2)\left(1+\frac{4m_Z^4}{s^2}\right)\right)\right]$$

$$= i\bar{v}(p_2)\,u(p_1)\left[\frac{2m_f}{v^2}\,\frac{1}{s-m_h^2}\left(\frac{s}{2} - m_Z^2 - \frac{2m_Z^4}{s} - \frac{4m_Z^6}{s^2}\right)\right]$$

Use $m_Z^2 = \frac{m_W^2}{c_W^2} = \frac{g^2v^2}{4c_W^2} \Rightarrow \frac{1}{v^2} = \frac{g^2}{4c_W^2\,m_Z^2}$

As a reminder, the g_V + g_A couplings of SM fermions are:

	g_V	g_A
u	$\frac{g}{c_W}\left(\frac{1}{4} - \frac{2}{3}s_W^2\right)$	$\frac{g}{c_W}\left(-\frac{1}{4}\right)$
d	$\frac{g}{c_W}\left(\frac{-1}{4} + \frac{1}{3}s_W^2\right)$	$\frac{g}{c_W}\left(\frac{1}{4}\right)$
ν	$\frac{g}{c_W}\left(\frac{1}{4}\right)$	$\frac{g}{c_W}\left(\frac{-1}{4}\right)$
e	$\frac{g}{c_W}\left(\frac{-1}{4} + s_W^2\right)$	$\frac{g}{c_W}\left(\frac{1}{4}\right)$

Gauge only

$$\sum_{spins} |M|^2 = \mathrm{Tr}\Big\{ \Big[\frac{1}{(t-m_Z^2)} \frac{1}{m_V^2} \left(\not p_4 - \frac{2m_V^2}{s}\not p_3\right)(g_V + g_A\gamma^5) \left(\not p_1 - \not p_3 + m\right) \left(\not p_3 - \frac{2m_V^2}{s}\not p_4\right)(g_V + g_A\gamma^5)$$

$\sum u\bar u = \not p + m$

$\sum v\bar v = \not p - m$

$$+ \frac{1}{(u-m_Z^2)} \frac{1}{m_V^2} \left(\not p_3 - \frac{2m_V^2}{s}\not p_4\right)(g_V + g_A\gamma^5) \left(\not p_1 - \not p_4 + m\right) \left(\not p_4 - \frac{2m_V^2}{s}\not p_3\right)(g_V + g_A\gamma^5) \Big]$$

$$\left[\not p_1 + m_1\right]$$

$$\Big[\frac{1}{t-m_Z^2} \frac{1}{m_V^2} \left(\not p_3 - \frac{2m_V^2}{s}\not p_4\right)(g_V + g_A\gamma^5) \left(\not p_1 - \not p_3 + m\right) \left(\not p_4 - \frac{2m_V^2}{s}\not p_3\right)(g_V + g_A\gamma^5)$$

$$+ \frac{1}{u-m_Z^2} \frac{1}{m_V^2} \left(\not p_4 - \frac{2m_V^2}{s}\not p_3\right)(g_V + g_A\gamma^5) \left(\not p_1 - \not p_4 + m\right) \left(\not p_3 - \frac{2m_V^2}{s}\not p_4\right)(g_V + g_A\gamma^5) \Big]$$

$$\left[\not p_2 - m_1\right] \Big\}$$

When we include Higgs diagrams, must add

$$\frac{-1}{s-m_h^2}\left(\frac{2m_f}{v^2}\right)\left(\frac{s}{2} - m_Z^2 - \frac{2m_Z^4}{s} - \frac{4m_Z^6}{s^2}\right) \text{ as extra term}$$

in each $\left[\frac{1}{t-m_Z^2} \cdots + \frac{1}{u-m_Z^2}\right]$ bracket.

Isolating $\frac{1}{m_Z^4}$ terms in $|M|^2$, we get a gauge contribution

$$\sum_s |M|^2 \supset \frac{-8g_A^2 s}{tu\, m_Z^4}\left(6g_V^2 m_Z^2 tu + g_A^2\left(m_1^2(s^2 + (t-u)^2) + m_Z^2 tu\right)\right)$$

and a Higgs-gauge + $(\text{Higgs})^2$ term of

$$\sum_s |M|^2 \supset m_1^2\left(\frac{g^4 stu + 8c_W g^2 g_A^2 (s+t-u)(s-t+u)(t+u)}{8c_W^4 tu}\right)$$

The problematic term from the gauge contributions is the middle term, since the others will cancel with the $\mathcal{O}\left(\frac{1}{m_Z^2}\right)$ terms we neglected.

So, we get

$$\sum_s |\mathcal{M}|^2_{tot}\Big|_{\frac{1}{m_Z^4}} = \frac{-8 g_A^4 s\, m_1^2 (-s^2 + (t-u)^2)}{t u\, m_Z^4} + \frac{m_1^2 \left(g^4 s t u + 8 c_W^2 g^2 g_A^2 (s+t-u)(s-t+u)(t+u)\right)}{8 c_W^4 t u}$$

Using $g_A = \frac{1}{4 c_W}$, this simplifies to

$$= g^4 \frac{m_1^2 (s+t+u)\,(s^2 - 2(t-u)^2 + s(t+u))}{32\, c_W^4\, t u}$$

Since $s+t+u = 2m_1^2 + 2m_Z^2$, the divergence for $s \to \infty$ is again resolved!

Consider the predicted coincidence required between g_A and the Yukawa coupling:

Higgs diagram $\propto \frac{y_f}{\sqrt{2}} \cdot \frac{2 m_Z^2}{v}$

(Axial) Gauge diagrams $\propto g_A^2 \cdot m_1$ ← from fermion propagator

We can match these by

$$g_A^2 m_1 \approx \frac{g^2}{c_W^2} \cdot y_1 v = \frac{g^2 v^2}{c_W^2} \frac{y_1}{v} = m_Z^2 \frac{y_1}{v}$$

And so, unitarity again <u>requires</u> this coincidence of axial-vector + Yukawa couplings.