QFT 3.

The Standard Model and Electroweak Theory

Lecture 4.

November 18, 2020.

Last time: Gauge currents + g_V vs. g_A couplings.
Unitarity + EWSB

Today: Finish unitarity discussion of WW scattering + "no-lose" theorem
$f\bar{f} \to WW / ZZ$ discussion + unitarity violation
Collider physics @ tree-level: Finding the Higgs boson

From the end of the last lecture, we discussed the tree-level calculation of $W^+W^- \to W^+W^-$.

This has s-channel $\gamma + Z$ diagrams, t-channel $\gamma + Z$ diagrams, and the 4 pt. gauge vertex diagram. There are also s-channel + t-channel Higgs diagrams.

We can simplify the calculation by approximating the polarization vectors as mostly longitudinal (aligned with momentum):

$$\epsilon_{1L}^\mu \approx \frac{p_1^\mu}{m_W} - \frac{2m_W}{s} p_2^\mu \qquad \text{(Ref. 1303.6335).}$$

$$\epsilon_{2L}^\mu \approx \frac{p_2^\mu}{m_W} - \frac{2m_W}{s} p_1^\mu$$

Then,

$$i\mathcal{M}_{s,\gamma+Z} = -i g^2 \left(\frac{s_W^2}{s} + \frac{c_W^2}{s - m_Z^2} \right) \Big((p_1 - p_2)_\mu \epsilon_1 \cdot \epsilon_2 + 2p_2 \cdot \epsilon_1 \epsilon_{2\mu} - 2p_1 \cdot \epsilon_2 \epsilon_{1\mu} \Big) \times \Big((p_4 - p_3)^\mu \epsilon_3^* \cdot \epsilon_4^* + 2p_3 \cdot \epsilon_4^* \epsilon_3^{*\mu} - 2p_4 \cdot \epsilon_3^* \epsilon_4^{*\mu} \Big)$$

$$i\mathcal{M}_{t,\gamma+Z} = i g^2 \left(\frac{s_W^2}{t} + \frac{c_W^2}{t - m_Z^2} \right) \Big(-(p_1 + p_3)_\mu \epsilon_1 \cdot \epsilon_3^* + 2p_1 \cdot \epsilon_3 \epsilon_{1\mu} + 2p_3 \cdot \epsilon_1 \epsilon_{3\mu}^* \Big) \Big((p_2 + p_4)^\mu \epsilon_2 \cdot \epsilon_4^* - 2p_4 \cdot \epsilon_2 \epsilon_4^{*\mu} - 2p_2 \cdot \epsilon_4^* \epsilon_2^\mu \Big)$$

$$i\mathcal{M}_4 = ig^2\left(2\epsilon_1\cdot\epsilon_4^*\,\epsilon_2\cdot\epsilon_3^* - \epsilon_1\cdot\epsilon_2\,\epsilon_3^*\cdot\epsilon_4^* - \epsilon_1\cdot\epsilon_3^*\,\epsilon_2\cdot\epsilon_4^*\right)$$

$$i\mathcal{M}_{s,h} = -i\,\epsilon_1\cdot\epsilon_2\,\epsilon_3^*\cdot\epsilon_4^*\quad\frac{g^2 m_W^2}{s-m_h^2}$$

$$i\mathcal{M}_{t,h} = -i\,\epsilon_1\cdot\epsilon_3^*\,\epsilon_2\cdot\epsilon_4^*\quad\frac{g^2 m_W^2}{t-m_h^2}$$

[See WW scattering calculation note for details.]

Applying the longitudinal approximation, the sum of the gauge diagrams becomes

$$i\mathcal{M}_{\substack{tot\\gauge}} = \frac{-ig^2}{4m_W^4}\left(s^2+2st+t^2-su-tu-2u^2\right)$$
$$+\frac{ig^2}{4s\,m_W^2}\left(7s^2-8t(t+u)-s(t+6u)\right) + \mathcal{O}(m_W^0)$$

Using $s+t+u=4m_W^2$, the first term becomes

$$-\frac{ig^2}{m_W^4}\left(4m_W^4-3m_W^2u\right),$$

which ensures the $\mathcal{O}\left(\frac{E^4}{m_W^4}\right)$ growth vanishes.

The ~~re~~remaining $\mathcal{O}\left(\frac{E^2}{m_W^2}\right)$ growth is

$$ig^2\frac{3u}{m_W^2} + \frac{ig^2}{4sm_W^2}\left(7s^2-8t(t+u)-s(t+6u)\right)$$
$$= \frac{ig^2}{m_W^2}\,\frac{\left(4m_W^2(7s-8t)-su\right)}{4s}$$
$$\approx -\frac{ig^2\,u}{4m_W^2} + \mathcal{O}(m_W^0).$$

This growth is deadly, since as $E\to\infty$, the matrix element grows unbounded.

Fortunately, this growth is cancelled when we include the Higgs diagrams:

$$i\mathcal{M}_{tot,\,h} = \frac{-ig^2}{4m_W^2}(s+t) - \frac{ig^2}{2m_W^2}m_h^2 + \mathcal{O}(m_W^0, m_h^0)$$

Adding the gauge amplitudes, we get

$$i\mathcal{M}_{tot} = \frac{-ig^2}{4m_W^2}\left(4m_W^2\right) - \frac{ig^2 m_h^2}{2m_W^2} + \mathcal{O}(m_W^0, m_h^0)$$

$$= -ig^2 - \frac{ig^2 m_h^2}{2m_W^2} + \mathcal{O}(m_W^0, m_h^0)$$

So, there is no $\mathcal{O}\left(\frac{E^2}{m_W^2}\right)$ growth in the amplitude anymore. We see that the mass parameter that replaces the $\mathcal{O}\left(\frac{E^2}{m_W^2}\right)$ growth is now the Higgs mass! Unitarity (perturbatively) at tree level requires that we cannot make the Higgs mass too high. In the language of Lagrangian parameters, we can trade

$$m_h^2 = 2\lambda v^2, \quad m_W^2 = \frac{g^2v^2}{4}, \quad \text{so}$$

$$-ig^2 \cdot \frac{2\lambda v^2}{\left(\frac{g^2v^2}{2}\right)} = -i4\lambda.$$

Thus, making the Higgs too heavy (for fixed EW scale) requires sending $\lambda \to \infty$, which will violate perturbative unitarity. This was the argument in Lee, Quigg, + Thacker (1977) that established a "no-lose" theorem for a collider at energies $\approx \mathcal{O}(1) - \mathcal{O}(4\pi) \cdot v \approx \mathcal{O}(1\ \text{TeV})$. A perturbative Higgs boson or some non-perturbative resolution to the unitarity violation in WW scattering must be encountered.

Comment: A precise bound can be obtained by using partial wave decomposition. This is essentially a Legendre polynomial decomposition of the amplitude + ensuring the prefactor coefficients in this series expansion are sufficiently small to satisfy unitarity. For details, see Schwartz, Sec. 24.1.5 and Sec. 29.2.3

Comment: This calculation has nothing to do with any matter content! We will address SM fermions next.

Comment: This calculation can be summarized by the statement that Lagrangian parameters must be chosen self-consistently, and you should be wary of exercising more parameter freedom than actual parameters in your model (i.e. choosing λ + m_h^2 separately). As an illustration, the moment you allow a deviation in the Higgs coupling to vector bosons, the unitarity problem becomes unresolved:

Ex. We assign a coupling reweighting factor of K_V for the Higgs couplings to W^+W^-, with $K_V = 1$ as the SM case.

$$\text{Then } i\mathcal{M}_{tot} = \frac{-ig^2}{4m_W^2}(s+t)K_V^2 - \frac{ig^2 m_h^2}{2m_W^2}K_V^2 - \frac{ig^2 u}{4m_W^2}$$

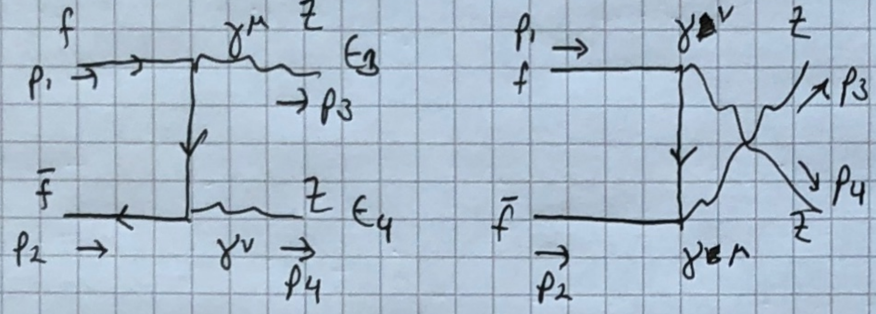
$$= \frac{-ig^2(4m_W^2 - u)K_V^2}{4m_W^2} - \frac{ig^2 u}{4m_W^2} - \frac{ig^2 m_h^2 K_V^2}{2m_W^2}$$

$$= -ig^2K_V^2 - \frac{ig^2 u}{4m_W^2}(1-K_V^2) - \frac{ig^2 m_h^2 K_V^2}{2m_W^2}$$

The scale of unitarity violation is determined by the $\frac{u}{m_W^2}(1-K_V^2)$ growth.

Now, we can analyze the SM fermions & unitarity violation. It turns out that there is a very similar behavior that restricts the axial-vector coupling and the mass/Yukawa of a given fermion. We start with $f\bar{f} \to ZZ$ scattering.

Gauge diagrams



See $ff\,VV$ scattering notebook for details.

Higgs diagram