Lecture 15

QFT III: The Standard Model + EW Theory

Felix Yu

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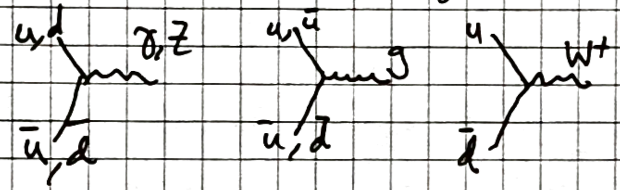
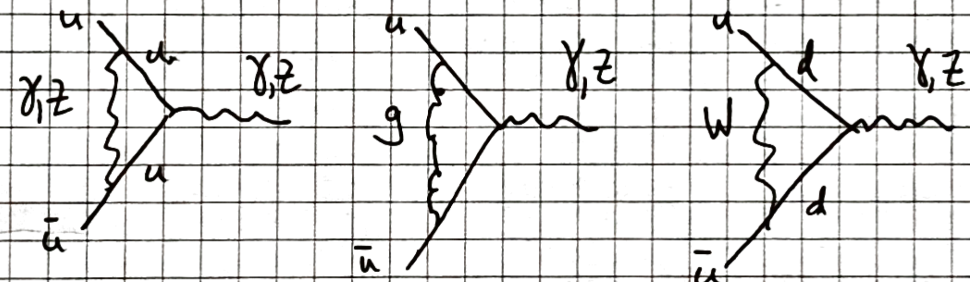
Last time: FCNCs + tree-level vs. loop-level SM

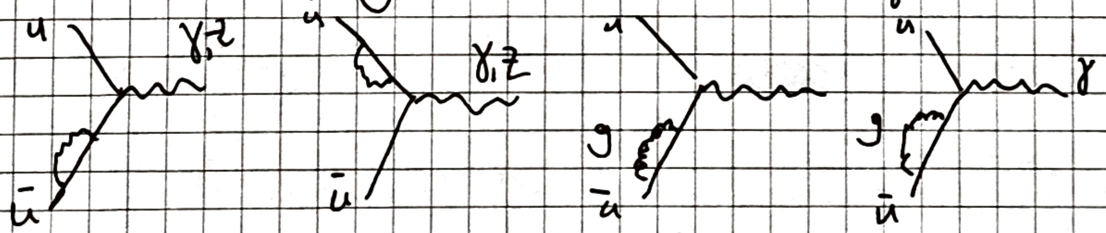
Today: Mixed 1-loop corrections + RG running.
EW Oblique corrections

Full set of u, d gauge one-loop corrections.



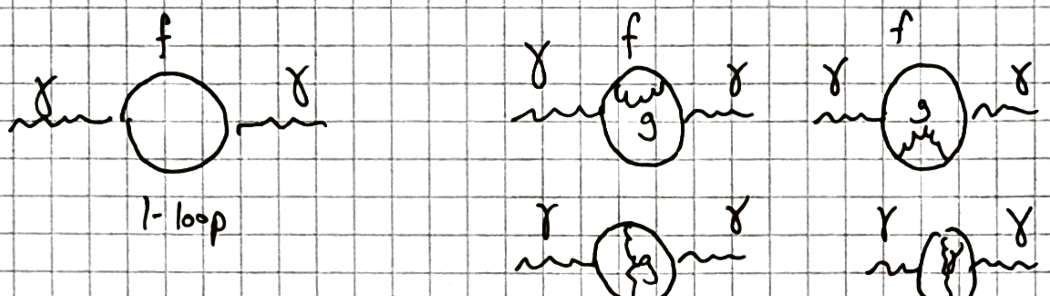


UV divergence needs to cancel with counterterm of tree-level coupling and also external leg corrections:



These are related to the β-fcns. of the corresponding gauge fcns. It turns out that the mixed gauge coupling diagrams only contribute to the β-fcns. of the tree-level interaction's gauge coupling at 2-loop order, which can be most easily seen by the wavefunction renormalization of the gauge bosons.

For QCD and QED, for example,



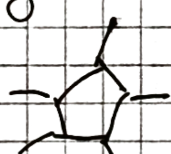
For $G_1 \times G_2$ gauge groups following D.R.T. Jones, PRD 25, 2 (1982),

$$\beta_{g_1} = \frac{1}{16\pi^2} g_1^3 \left[\frac{2}{3} T(R_1)\, d(R_2) + \frac{1}{3} T(S_1)\, d(S_2) - \frac{11}{3} C_2(G_1) \right]$$
$$+ \frac{1}{(16\pi^2)^2} g_1^5 \Big\{ \left[\frac{10}{3} C_2(G_1) + 2 C_2(R_1) \right] T(R_1)\, d(R_2)$$
$$+ \left[\frac{2}{3} C_2(G_1) + 4 C_2(S_1) \right] T(S_1)\, d(S_2)$$
$$- \frac{34}{3} \left[C_2(G_1) \right]^2 \Big\}$$
$$+ \frac{1}{(16\pi^2)^2} g_1^3 g_2^2 \left[2 C_2(R_2)\, d(R_2)\, T(R_1) + 4 C_2(S_2)\, d(S_2)\, T(S_1) \right]$$

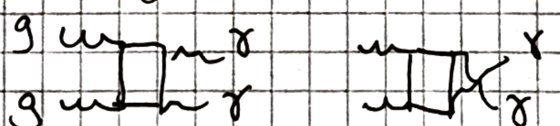
The 3-loop results for the SM β-fcns can be found in Mihaila, Salomon, Steinhauser [1201.5868].

Aside: ① diagrams like $gg\gamma$ or $ZZ\gamma$ vanish from anomaly cancellation. See Chirality & Gauge Theories lecture series by FY. Also Chap. 19, P&S.

② Diagrams with 5 legs are necessarily convergent.

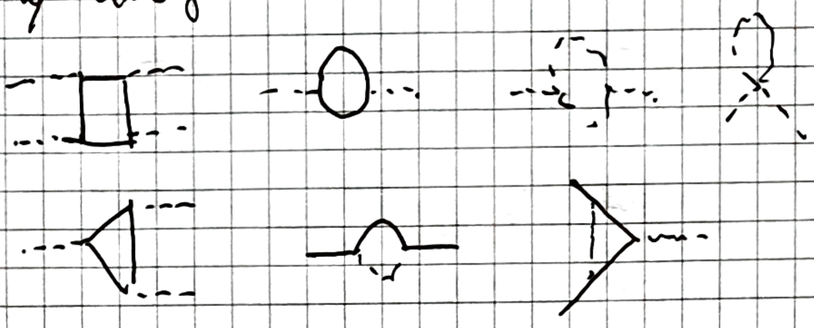
 $\sim \int d^4k \, \frac{1}{(k)^5}$

③ Diagrams with 4 external vectors (not all photons) are higher dim. operators. ⇒ Necessarily finite.



In fact, the set of 1-loop divergent graphs is finite.

Possibly divergent:



Each has a corresponding renormalizable tree-level operator that can have a CT.

Anomaly cancellation condition:

For chiral representations, loop integral is generally not invariant under shift of loop momenta. Instead, set of fermions need to have a prescribed set of charges & reps s.t. the shift, after summing over all fermions, gives no net change.

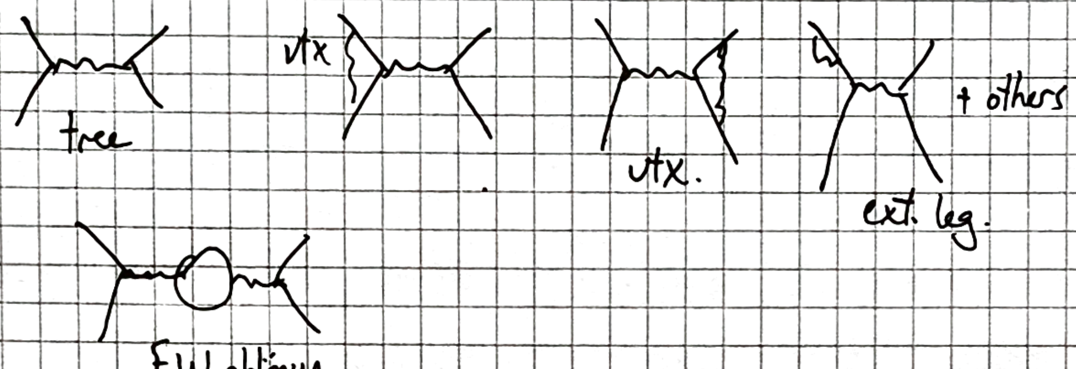
Aside from FCNCs & penguin diagrams, the other set of 1-loop diagrams are the electroweak oblique corrections, which give Q^2-dependent form factors for the interactions of EW gauge bosons.

These are generally studied in flavor-conserving processes, since the loop is saturated by the gauge bosons.

Recall: $J_{W^+} \otimes J_{W^-}$ = Fermi interaction, dim.-6 op.

So, we can intuit that EW oblique corrections are subsumed by modern language of dim. 6 EFT.

In words, EW oblique corrections explore finite Q^2 form factors of EW bosons away from pole values, since pole behavior of EW bosons is controlled by 1-loop renormalization conditions.



Experimentally, this is motivated because the initial state was e^+e^- at $2E_f \Rightarrow$ can control $\sqrt{s}$.

See Schwartz, Sec. 31.12.

Peskin-Takeuchi:

$$S = \frac{4c^2s^2}{\alpha_e}\left[\frac{\Pi^{new}_{ZZ}(m_Z^2) - \Pi^{new}_{ZZ}(0)}{m_Z^2} - \frac{c^2-s^2}{cs}\frac{\Pi^{new}_{Z\gamma}(m_Z^2)}{m_Z^2} - \frac{\Pi^{new}_{\gamma\gamma}(m_Z^2)}{m_Z^2}\right]$$

$$T = \frac{1}{\alpha_e}\left[\frac{\Pi^{new}_{WW}(0)}{m_W^2} - \frac{\Pi^{new}_{ZZ}(0)}{m_Z^2}\right]$$

$$U \equiv \frac{4s^2}{\alpha_e}\left[\Pi^{new}_{WW}(m_W^2) - \Pi^{new}_{WW}(0) - \frac{c}{s}\frac{\Pi^{new}_{Z\gamma}(m_Z^2)}{m_Z^2} - \frac{\Pi^{new}_{\gamma\gamma}(m_Z^2)}{m_Z^2}\right] - S$$

$c, s = c_\theta, s_\theta$, $\alpha_e = \frac{e^2}{4\pi}$, Π^{new} = only new particles.

$S = T = U = 0$ in SM.

Note: Always evaluate at m^2_{pole} or 0.

The easiest EW oblique parameter to explain is T:

$\Pi_{WW}(0) = m_W^2$ since $q^2 \to 0$ is location of pole

$\Pi_{ZZ}(0) = m_Z^2$

T evaluates corrections to m_W^2 + m_Z^2 ~~for~~ from new physics, looking for a violation of custodial symmetry.

Tree-level relation: $m_W^2 = m_Z^2 c_W^2$

From new physics, $\tilde{m}_W^2 = m_W^2 + \delta m_W^2$, $\tilde{m}_Z^2 = m_Z^2 + \delta m_Z^2$

+ test $\delta m_W^2 = \delta m_Z^2 c_W^2$ (or $c_W^2 + \delta c_W^2$)

To understand theoretical corrections in the context of measurements, would need to have a dedicated discussion on input measurements vs. derived parameters.

The S-parameter evaluates WFR at m_Z vs. at $m=0$

Essentially, how Z-like is photon @ m_Z + how photon-like is Z @ $m=0$.

In modern language,

pp collider $\Rightarrow$ cannot control $\sqrt{s}$ much + instead focus on entire $\frac{d\sigma}{dq^2}$ distribution in EW physics, where the natural language is dim-6 SM EFT.

$$O_S = \frac{1}{\Lambda^2} H^\dagger \sigma^i H \, W^i_{\mu\nu} B^{\mu\nu}$$

$$O_T = \frac{1}{\Lambda^2} \left| H^\dagger D_\mu H \right|^2$$