Lecture 10.

QFT 3: The Standard Model and EW Theory.

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Last time: IR divergences and radiation of soft and/or collinear gauge bosons.

Today: Concept of jets from quarks + gluons to collection of hadrons

Typical collider objects + detection efficiencies

Trigger, acceptances, + general collider terminology

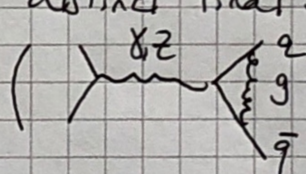
The story so far:

We can readily make an analogy between the hard scattering process of $e^+e^- \to q\bar{q}$ and the QED process of $e^+e^- \to \mu^+\mu^-$.

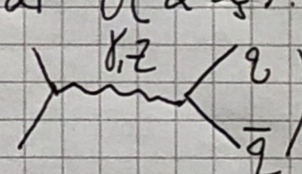
When we want to understand the $\mathcal{O}(\alpha^2\alpha_s)$ correction to $e^+e^- \to$ hadrons, we study the $\mathcal{O}(\alpha^3)$ correction in $e^+e^- \to \mu^+\mu^-$.

We had two distinct final states at $\mathcal{O}(\alpha^2\alpha_s)$:

$q\bar{q}$: $\dfrac{d\sigma}{d\Omega} \sim$ (

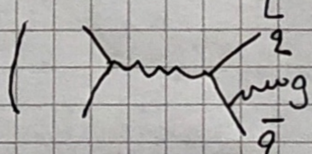


$\times$

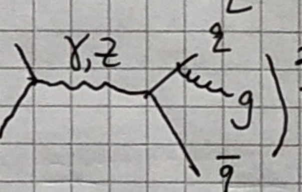


) $\to \mathcal{O}(\alpha^2\alpha_s)$

$q\bar{q}g$: $\dfrac{d\sigma}{d\Omega} \sim$ (



$+$



$)^2 \to \mathcal{O}(\alpha^2\alpha_s)$

From gauge invariance, we know that all processes (that are 1PI) at a given fixed order in coupling power should be included in order to get a gauge invariant result. So even though the final states are different, the sum is gauge invariant.

The sum has an additional benefit that it is finite, since the IR divergence (for $\mu^2 \to 0$) is cancelled! In other words, final states are always ambiguous by the presence of soft or collinear radiation.

We had differential cross sections of

$\mu^+\mu^-$ production/scattering of electron w/ momentum p to electron w/ momentum p' from potential
$$\frac{d\sigma}{d\Omega} \simeq \left(\frac{d\sigma}{d\Omega}\right)_0 \Bigg(1 - \underbrace{\frac{\alpha}{\pi} \log\left(\frac{-q^2}{m^2}\right) \log\left(\frac{-q^2}{\mu^2}\right)}_{\text{Sudakov double log}} + \mathcal{O}(\alpha^2)\Bigg)$$

$q^2 < 0 \Rightarrow \mathcal{O}(\alpha)$ correction is negative & infinite

bremsstrahlung process
$$\frac{d\sigma}{d\Omega} \simeq \left(\frac{d\sigma}{d\Omega}\right)_0 \left(\frac{\alpha}{\pi} \log\left(\frac{-q^2}{m^2}\right) \log\left(\frac{-q^2}{\mu^2}\right) + \mathcal{O}(\alpha^2)\right)$$

Sum of these processes is independent of μ^2 (fictitious photon mass) and <u>finite</u>!

The measured cross section is thus the sum, where the detector resolution on observable photons E_γ replaces the μ^2 cutoff.

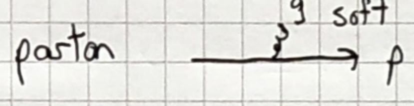
$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{measured}} \simeq \left(\frac{d\sigma}{d\Omega}\right)_0 \left(1 - \frac{\alpha}{\pi} f_{IR}(q^2) \log\left(\frac{-q^2 \text{ or } m^2}{E_\gamma^2}\right) + \mathcal{O}(\alpha^2)\right)$$

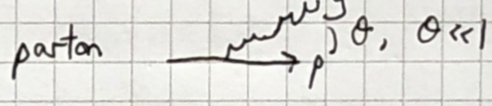
Note now that $\log\left(\frac{-q^2}{E_\gamma^2}\right)$ or $\log\left(\frac{m^2}{E_\gamma^2}\right)$ is a physical enhancement.

This enhanced IR divergence & cancellation carries over from QED to QCD after we replace photons by gluons and color factors.

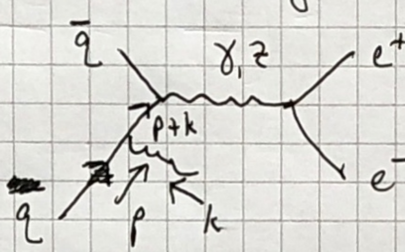
So, we will always consider, for example, $\sigma(e^+e^- \to q\bar{q})$ and $\sigma(e^+e^- \to q\bar{q}\, g_{\text{soft}})$ and $\sigma(e^+e^- \to q\bar{q}\, g_{\text{collinear}})$ together as one observable rate, leaving hard or wide angle gluons separate.

Why are the divergences soft &/or collinear?





Following lecture notes by Prestel, analyze $q\bar{q} \to e^+e^-$ (crossing symmetry reversal of $e^+e^- \to q\bar{q}$) & allow ISR radiation of gluon:



Matrix element has

$$u(p)\,\not{\epsilon}\,\frac{\not{p}+\not{k}}{(p+k)^2-m^2} \quad \cdots \Rightarrow \quad u(p)\,\frac{p\cdot\epsilon}{m^2+2p\cdot k-m^2}$$

$$\sim u(p)\,\frac{p\cdot\epsilon}{2p\cdot k}$$

Now, $p\cdot k = (E_q E_g - \vec{p}\cdot\vec{k})$

$= E_g\,(E_q - |\vec{p}|\cos\theta)$ ← angle between $\vec{p}$ & $\vec{k}$ Use $E_g = |\vec{k}|$ since gluon is massless

For massless quark,

$= E_g E_q (1-\cos\theta)$

So, $u(p)\,\frac{p\cdot\epsilon}{2p\cdot k} \sim u(p)\,\frac{p\cdot\epsilon}{E_g E_q (1-\cos\theta)}$

gives a <u>soft</u> divergence as $E_g \to 0$, independent of θ

& a <u>collinear</u> divergence as $\theta \to 0$, independent of E_g

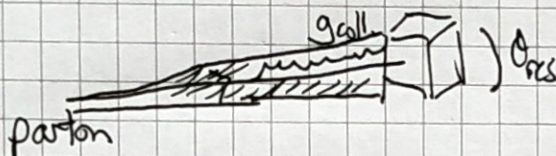
& a <u>soft</u>-<u>collinear</u> divergence as $E_g \to 0$ <u>and</u> $\theta \to 0$.

★ Always get soft divergence before collinear divergence.

This is the basis for SCET, soft-collinear effective theory.

In principle, massive quarks ⇒ $E_q \neq |\vec{p}|$ & regulates exact collinear divergence. Also, massive gauge bosons (W, Z) will regulate/cutoff the soft divergence.

In practice, the collinear divergence is regulated by having a finite angular resolution detector element that measures a given parton.



collinear gluon is indistinguishable within finite $\theta_{res.}$

And the soft divergence is resolved by the threshold minimum energy on photons.

So, to reiterate, the IR-safe cross section observables must allow additional soft and/or collinear radiation. Finite calculations are then divided by multiplicity of hard / wide-angle radiation:

$q\bar{q}$ + soft/collinear = 2 jets exclusive

$q\bar{q}$ + 1 hard g + soft/collinear = 3 jets exclusive

$q\bar{q}$ + 2 hard g + soft/collinear = 4 jets inclusive.

where we always retain an implicity detector threshold on energy + angular resolution.

The collider object that encompasses these restrictions is the <u>jet</u> : collection of hadrons that sums over all hadrons within some finite angle + some energy threshold for these hadrons. Common jet algorithms to cluster the hadrons together are

Cambridge / Aachen: angular ordering algorithm

$$\left.\begin{matrix} k_T \\ \text{anti-}k_T \end{matrix}\right\} \text{ soft-ordering algorithms}$$

Progression from high-energy parton to jet:

High energy quark or gluon radiates gluons and/or splits into $q\bar{q}$ pairs.

Then hadronize into mesons + baryons via color recombination (pulling colored particles out of the vacuum) at $\tau \sim \frac{1}{\Lambda_{QCD}}$. These hadrons then decay or live long enough to interact with a detector to be registered as energy deposits + momenta hits.

The jet finding algorithm clusters these 4-vectors into a single jet 4-vector according to the algorithm. C/A, for example, chooses the highest energy hadron as a seed axis + adds in all hadrons within some angular radius R, which is user-defined.

k_T + anti-k_T choose neighboring hadrons by their momentum weight relative to the seed axis.

We end up with a set of IR safe collider observables, aka jets.