08.128.742 Quantum Field Theory III
The Standard Model and Electroweak Theory

Homework set 2

Due November 26, 2020; e-mail (photo or scan) to yu001@uni-mainz by start of discussion session
Please note how long it took you to solve each problem

2-1, 50 pts. A. Draw the Feynman diagram for tree-level muon decay and write the matrix element. B. Expand the W propagator to leading powers of p^2/m_W^2, where p^2 is the momentum transfer carried by the W boson. C. Match the matrix element expression to the dimension-6 Fermi interaction and demonstrate that the Fermi effective operator gives the same leading order result at $1/m_W^2$ accuracy. D. You can also generate the full Fermi theory by taking the $J^\mu_{W^+}$ and $J^\mu_{W^-}$ currents and multiplying them together (keep in mind the necessary CKM factors for quarks). Why does this work? E. Write out the tree-level charged-current decays of a b quark using Fermi theory. If the b quark combines with a spectator u quark to form a B^+ meson, what final states do these charged-current decays predict and what is the rough relative rate between these different final states? (If you like, you can compare to the table of B^+ decay modes from the PDG.) Why is the charged current interaction dominant for b-quark decays?

2-2, 50 pts. One category of dimension-6 effective operators in the SMEFT (Standard Model Effective Field Theory) is the augmentation of any dimension-4 SM Lagrangian term by $H^\dagger H/\Lambda^2$, where H is the Higgs doublet and Λ is a scale of new physics. A. Take the SM down-quark Yukawa term and its $H^\dagger H/\Lambda^2$ dimension-6 EFT term,

$$\mathcal{L} = y_d \overline{Q}_L H d_R + y'_d \frac{H^\dagger H}{\Lambda^2} \overline{Q}_L H d_R + \text{ h.c..} \tag{1}$$

Calculate the down quark mass matrix including the EFT contribution and apply the usual diagonalization procedure from the (unchanged) global symmetries on the fermion gauge eigenstates. B. What are the new Yukawa couplings of the Higgs boson to the down quarks? What requirements are needed on y_d and y'_d in order to have the Yukawa couplings be diagonal and CP conserving? (It is also interesting to think about the fine-tuning of fermion masses in the dim-6 SMEFT.) C. SM trivia: What leading dim-6 operators correspond to the Higgs-digluon coupling and the Higgs-diphoton couplings? Why do these operators not correspond to a gluon mass or a photon mass?