Move next week's lectures to mornings?

Proposal: Th 9:30-11:30 am (c.t.)
W 8-10 am (c.t.)

HW 3 discussion session tomorrow

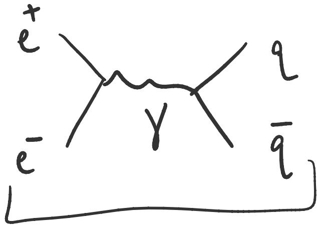
Today: Parton showers & collider objects.

In practice, because of fragmentation & hadronization, we do not measure individual quarks & gluons at colliders, but instead we measure <u>jets</u>, which are collections of hadrons (mesons & baryons) that arise from partonic cross sections with final state quarks & gluons.

Central question: What are the IR-safe cross sections for quark & gluon production?

Simplest process:

$e^+e^- \to$ hadrons, i.e. $e^+e^- \to q\bar{q}$ (via γ)



analogous to $e^+e^- \to \mu^+\mu^-$

(not $e^+e^- \to e^+e^-$)

Procedure: find exact analogous QED (-like) process & then reweight for $q\bar{q}$ final state.

$\sigma(e^+e^- \to \Sigma\, q_a \bar{q}_a)) = \sigma(e^+e^- \to \mu^+\mu^-) \cdot N_c Q^2$

... reweight for $q\bar{q}$ final state.

$$\sigma\left(e^+e^- \to \left(\sum q_f \bar{q}_f\right)\right) = \sigma_0\left(e^+e^- \to \mu^+\mu^-\right) \cdot N_c \sum Q_f^2$$

$\sigma_0\left(e^+e^- \to \mu^+\mu^-\right)$ = QED cross sections

N_c: color (summing over final states)

$\sum Q_f^2$: reweight by electric quark charge

$$\sigma_0 = \frac{4\pi\alpha^2}{3s}$$

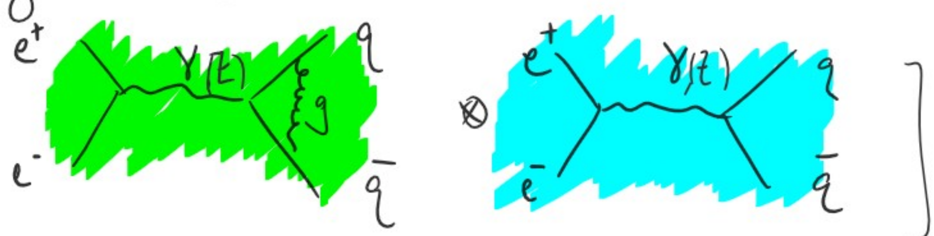
[Ignore masses of quarks + leptons.]

Simplest tree-level process, no obvious issue.

Consider leading correction $\underline{\mathcal{O}(\alpha^2\alpha_s)}$:

1PI Diagrams are:

$\mathcal{O}(\alpha^2\alpha_s) \sim$ [



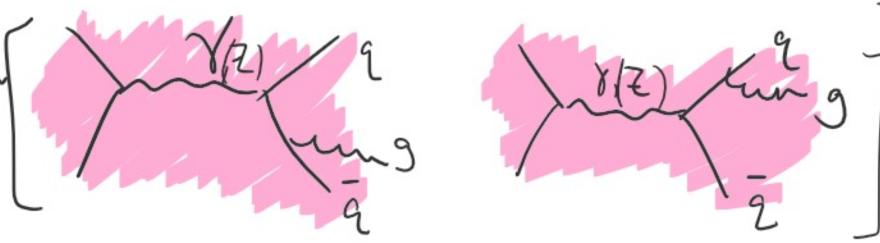
]

$\mathcal{M} \sim e^2$

$\mathcal{M} \sim e^2 g_s^2$

$\mathcal{M} \sim e^2 g_s$

$\mathcal{O}(\alpha^2\alpha_s) \sim$ [



]

These all contribute to the cross section at the same power counting of couplings.

So, a perturbative expansion would necessarily be constructed by

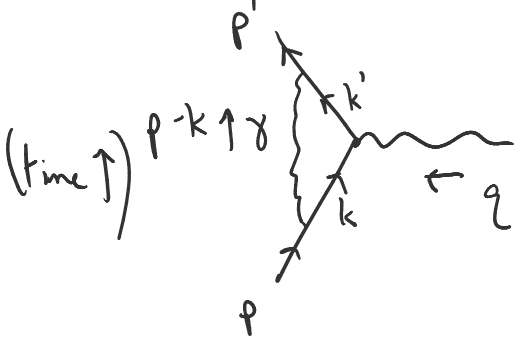
$$\sigma \sim \Big|\underbrace{\quad}_{\mathcal{O}(\alpha^2)}\Big|^2 + \underbrace{\overbrace{\left|\text{tree} \otimes \text{1-loop}\right|^2}^{\mathcal{O}(\alpha^2\alpha_s)}} + \overbrace{\left|\text{real rad.}\right|^2}^{\mathcal{O}(\alpha^2\alpha_s)} + \mathcal{O}(\alpha^2\alpha_s^2) + \mathcal{O}(\alpha^3\alpha_s^0) + \ldots$$

↳ diverges from IR divergence in vtx.

↳ seen the $\mathcal{O}(\alpha^3)$ calculation in QED.

Recall, $\sigma(e^+e^- \to \mu^+\mu^-) + \sigma(e^+e^- \to \mu^+\mu^-\gamma)$ in QED at $\mathcal{O}(\alpha^3)$.

Following P+S, Sec. 6.3.

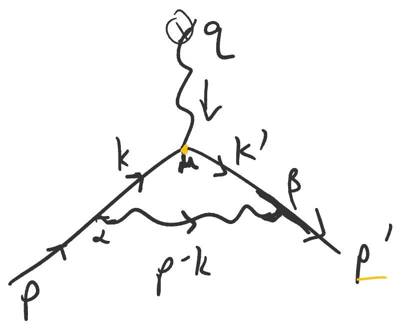


usual γ^μ vtx

Generally, $\Gamma^\mu(p,p') = \gamma^\mu F_1(q^2) + \dfrac{i\sigma^{\mu\nu}q_\nu}{2m_e} F_2(q^2)$

$F_1(q^2=0)$ = electric charge of the electron.

F_2 = magnetic charge



R_ξ is not necessary for QED since not SSB.

For $\Gamma^\mu = \gamma^\mu + \delta\Gamma^\mu$ (1-loop)

$\bar{u}(p')\,\delta\Gamma^\mu(p',p)\,u(p)$

$$= \int \frac{d^4k}{(2\pi)^4}\; \bar{u}(p')\,(-ie\gamma^\beta)\,\frac{i(\not{k}'+m_e)}{k'^2-m_e^2}\cdot(-ie\gamma^\mu)\cdot\frac{i(\not{k}+m_e)}{k^2-m_e^2}\cdot(-ie\gamma^\alpha)\cdot u(p)\,\underbrace{\frac{(-ig_{\alpha\beta})}{(p-k)^2}}$$

[$\times\ \epsilon_\mu(q)$ but leave off to keep uncontracted]

~~Lorentz~~ gauge choice.

Feynman, $\xi=1$ [eqn. 9.58]

$$\tilde{D}_F^{\mu\nu}(k) = \frac{-i}{k^2+i\epsilon}\left(g^{\mu\nu} - (1-\xi)\frac{k^\mu k^\nu}{k^2}\right)$$

[Break until 3:10]

$$= 2ie^2\int\frac{d^4k}{(2\pi)^4}\,\frac{\bar{u}(p')\left[\not{k}\gamma^\mu\not{k}' + m_e^2\gamma^\mu - 2m_e(k+k')^\mu\right]u(p)}{((k-p)^2+i\epsilon)\,(k'^2-m^2+i\epsilon)\,(k^2-m^2+i\epsilon)}$$

$$\frac{\qquad}{\left((k-p)^2+i\epsilon\right)\left(k'^2-m^2+i\epsilon\right)\left(k^2-m^2+i\epsilon\right)}$$

Usual Feynman params., dim. reg. (+ Pauli-Villars reg.) [for UV divs.]

$$\Delta \equiv -xyq^2+(1-z)^2m^2$$

$$\Rightarrow F_1(q^2) = 1+\frac{\alpha}{2\pi}\int_0^1 dx\,dy\,dz\,\delta(x+y+z-1)$$

$$\left[\log\left(\frac{m^2(1-z)^2}{m^2(1-z)^2-q^2xy}\right)+\frac{m^2(1-4z+z^2)+q^2(1-x)(1-y)}{m^2(1-z)^2-q^2xy+\mu^2z}\right.$$

$$\left.-\frac{m^2(1-4z+z^2)}{m^2(1-z)^2+\mu^2z}\right]+\mathcal{O}(\alpha^2)$$

$$F_2(q^2) = \frac{\alpha}{2\pi}\int_0^1 dx\,dy\,dz\,\delta(\ldots)\left[\frac{2m^2z(1-z)}{m^2(1-z)^2-q^2xy}\right]+\mathcal{O}(\alpha^2)$$

where we introduced a photon mass μ^2, otherwise problematic IR divergence for $m^2\to 0$, $q^2\to 0$.

Simplify, study physical consequence of this divergence.

$F_1(q^2)$ is divergent for $z=1$, $x=y=0$.

$$F_1(q^2)\sim\frac{\alpha}{2\pi}\int_0^1 dz\int_0^{1-z}dy\left[\frac{m^2(1-4z+z^2)+q^2(z+y)(1-y)}{m^2(1-z)^2-q^2y(1-z-y)+\mu^2z}\right.$$

$$\left.-\frac{m^2(1-4z+z^2)}{m^2(1-z)^2+\mu^2z}\right]$$

Send $z=1$, $x=y=0$ in numerators

$$F_1(q^2)\sim\frac{\alpha}{2\pi}\int_0^1 dz\int_0^{1-z}dy\left[\frac{-2m^2+q^2}{m^2(1-z)^2-q^2y(1-z-y)+\mu^2}\right.$$

$$+\,2m^2 \qquad\qquad \Big]$$

$$m^2(1-z)^2 - q^2 y(1-z-y) + \mu^2$$

$$+ \frac{2m^2}{m^2(1-z)^2+\mu^2}\Bigg]$$

Change variables: $y = (1-z)\xi, \quad w = (1-z)$

$$F_1(q^2) = \frac{\alpha}{4\pi}\int_0^1 d\xi \left[\frac{-2m^2+q^2}{m^2-q^2\xi(1-\xi)} \log\left(\frac{m^2-q^2\xi(1-\xi)}{\mu^2}\right) + 2\log\left(\frac{m^2}{\mu^2}\right)\right]$$

Schematically,

$$F_1(q^2) = 1 - \frac{\alpha}{2\pi} f_{IR}(q^2) \log\left(\frac{-q^2 \text{ or } m^2}{\mu^2}\right) + \mathcal{O}(\alpha^2)$$

$$f_{IR}(q^2) = \int_0^1 \left(\frac{m^2 - q^2/2}{m^2 - q^2\xi(1-\xi)}\right) d\xi \; - 1$$

Now, recall $F_1(q^2)$ multiplies γ^μ in the vtx fcn.

So, we can essentially study a "renormalized charge" contribution for the IR divergence by replacing $e \to F_1(q^2) \cdot e$.

$$\frac{d\sigma}{d\Omega} \simeq \left(\frac{d\sigma}{d\Omega}\right)_0 \left[\underset{\substack{\uparrow \\ \text{tree} \\ (F_1(q^2=0))}}{1} - \underset{\substack{\nearrow \\ \text{loop}}}{\frac{\alpha}{\pi} f_{IR}(q^2)} \log\left(\frac{-q^2 \text{ or } m^2}{\mu^2}\right) + \mathcal{O}(\alpha^2) \right]$$

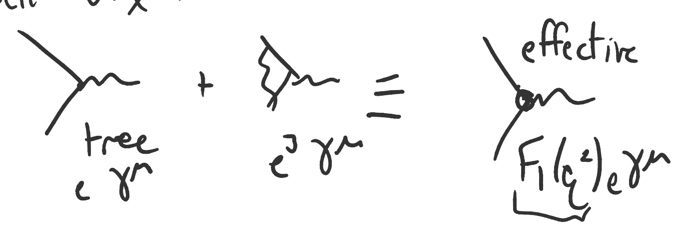
α

$\alpha^{3/2} \sim e^3$

cross section diverges for $\mu^2 \to 0$.

$(F_1)^2$

Overall vtx:



Evaluate $f_{IR}(q^2)$ as $-q^2 \to \infty$

$$\int^1 d\xi \quad \cdots q^2/2 \cdots$$

Evaluate $F_{IR}(q)$ as $q \to \infty$

$$\int_0^1 d\xi \, \frac{-q^2/2}{-q^2(\xi)(1-\xi)+m^2} \simeq \frac{1}{2}\int d\xi \, \frac{-q^2}{-q^2\xi+m^2} + \left(\begin{matrix}\text{equal for}\\ \xi \approx 1\end{matrix}\right)$$

$$= \log\left(\frac{-q^2}{m^2}\right)$$

$$\Rightarrow F_1(-q^2 \to \infty) = 1 - \frac{\alpha}{2\pi}\log\left(\frac{-q^2}{m^2}\right)\log\left(\frac{-q^2}{\mu^2}\right) + O(\alpha^2)$$

However, we're saved by the real radiation process. [not allowed to have $q^{\sqrt{3}}$ behavior]

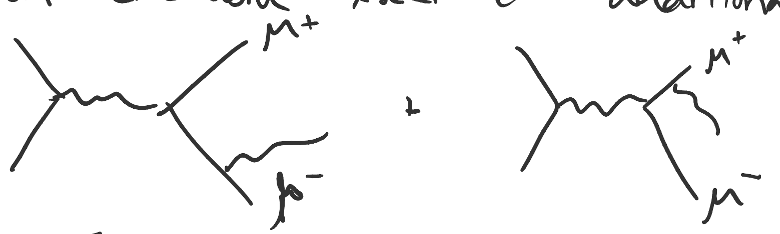
$$\frac{d\sigma}{d\Omega}\left(\bar{e}(p) \to \bar{e}(p') + \gamma\right) = \left(\frac{d\sigma}{d\Omega}\right)_0\left[\frac{\alpha}{\pi}\log\left(\frac{-q^2}{m^2}\right)\log\left(\frac{-q^2}{\mu^2}\right) + O(\alpha^2)\right]$$

$\frac{d\sigma}{d\Omega}\left(e^-(p) \to e^-(p') + n\gamma(k < E_\ell)\right)$ is then finite + exclusive.

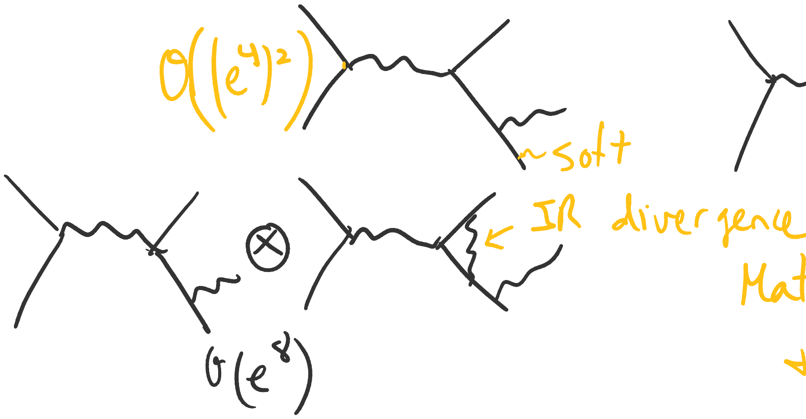
soft photons that are undetectable.

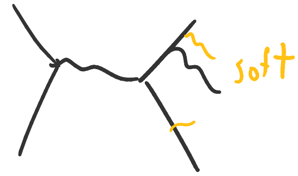
This is exactly reproduced for $e^+e^- \to q\bar{q}$ + $e^+e^- \to q\bar{q}g$.

1-photon exclusive xsec. (allow additional soft photons)



IR divergence here is truncated + goes into the 0-photon xsec.





Match again vtx corr.
+ soft photons even w/
some # of exclusive photons.

Final state radiation is the prediction for final state vertex correction.

Initial state radiation is also prediction for initial state vtx correction (gets into reweighting of PDFs).