Lecture 9.
QFT 3: The Standard Model + Electroweak Theory
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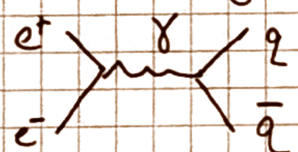
Today: Parton showers + collider objects.

In practice, because of fragmentation + hadronization, we do not measure quarks + gluons at colliders but instead only jets, which are collections of hadrons ($\approx$ mesons + baryons) that arise from partonic cross sections $\approx$ with final state quarks + gluons.

We begin with analyzing hadron production from e^+e^- initial state.

$e^+e^- \to q\bar{q}$ is analogous to $e^+e^- \to \mu^+\mu^-$, only s-channel diagram.



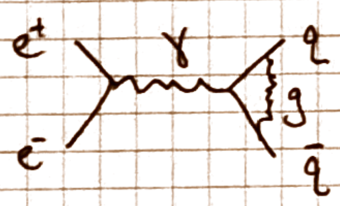
Only differences:

① Sum over final state colors
② Weight by electric charge.

Can derive $\sigma(e^+e^- \to \text{hadrons}) = \sigma_0 \cdot 3 \sum_f Q_f^2$,

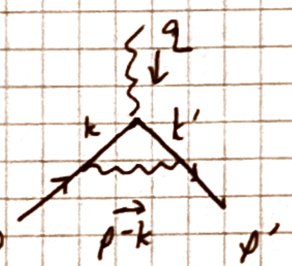
with $\sigma_0 = \dfrac{4\pi\alpha^2}{3s}$. (We will neglect quark + lepton masses.) At leading order, there is no obvious problem.

At subleading order, (either diagrams with an additional gluon or an additional photon), we have an important consideration.



has an IR divergence.

Recall QED: (P+S, Sec. 6.3).



Generally, $\Gamma^\mu(p',p) = \gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2m_e} F_2(q^2)$

$F_1(0)$ = electric charge of electron

F_1 + F_2 are electric & magnetic form factors, respectively.

For $\Gamma^\mu = \overset{\text{tree}}{\gamma^\mu} + \overset{\text{loop}}{\delta\Gamma^\mu}$

$$\bar{u}(p')\,\delta\Gamma^\mu(p',p)\,u(p)$$
$$= 2ie^2 \int \frac{d^4k}{(2\pi)^4}\, \frac{\bar{u}(p')\left[\not{k}\gamma^\mu \not{k}' + m_e^2\gamma^\mu - 2m_e(k+k')^\mu\right]u(p)}{\left((k-p)^2+i\epsilon\right)\left(k'^2-m^2+i\epsilon\right)\left(k^2-m^2+i\epsilon\right)}$$

Using Feynman parameters, $\Delta = -xyq^2 + (1-z)^2 m^2$, + subtracting off $F_1(q^2=0)$ to mimic the UV divergence cancellation from external leg corrections,

$$F_1(q^2) = 1 + \frac{\alpha}{2\pi}\int_0^1 dx\,dy\,dz\,\delta(x+y+z-1)$$
$$\left[\log\left(\frac{m^2(1-z)^2}{m^2(1-z)^2-q^2xy}\right) + \frac{m^2(1-4z+z^2)+q^2(1-x)(1-y)}{m^2(1-z)^2-q^2xy+\mu^2 z} - \frac{m^2(1-4z+z^2)}{m^2(1-z)^2+\mu^2 z}\right] + O(\alpha^2)$$

Here, μ^2 = fictitious photon mass.

$$F_2(q^2) = \frac{\alpha}{2\pi}\int_0^1 dx\,dy\,dz\,\delta(x+y+z-1)\left[\frac{2m^2 z(1-z)}{m^2(1-z)^2-q^2xy}\right]$$

Note that $F_2(q^2)$ has no IR divergence (nor any UV divergence). In fact,

$$F_2(q^2=0) = \frac{\alpha}{2\pi}\int_0^1 dx\,dy\,dz\; \delta(x+y+z-1)\,\frac{2z}{1-z}$$

$$= \frac{\alpha}{\pi}\int_0^1 dz\int_0^{1-z} dy\,\frac{z}{1-z} = \frac{\alpha}{2\pi},$$

which is the famous Schwinger term.

$$a_e = \frac{g-2}{2} = \frac{\alpha}{2\pi}.$$

So, we focus on the F_1 form factor: we want to isolate the terms that are sensitive to the IR divergence.

The non-log terms in $F_1(q^2)$ have the $\mu^2\to 0$ divergence: to see this explicitly, we can evaluate $F_1(q^2)$ at $q^2=0$ in the original $F_1(q^2)$

$$\Rightarrow\; F_1(q^2=0) \sim \frac{\alpha}{2\pi}\int_0^1 dx\,dy\,dz\;\delta(\ldots)\,\frac{1-4z+z^2}{m^2(1-z)^2}$$

$$= \frac{\alpha}{2\pi}\int_0^1 dz\int_0^{1-z} dy\,\frac{1-4z+z^2}{m^2(1-z)^2}$$

$$= \frac{\alpha}{2\pi}\int_0^1 dz\,\frac{1-4z+z^2}{m^2(1-z)}$$

$$= \frac{\alpha}{2\pi}\int_0^1 dz\,\frac{-2+(1-z)(3-z)}{m^2(1-z)}$$

$$= \frac{\alpha}{2\pi}\int_0^1 dz\,\frac{-2}{m^2(1-z)} + \text{finite}$$

So we get a logarithmic divergence from the singularity as $z\to 1$.

Going back to the full expression + isolating the $\mu^2\to 0$ IR divergence,

$$F_1(q^2) \sim \frac{\alpha}{2\pi}\int_0^1 dx\,dy\,dz\;\delta(x+y+z-1)\left[\frac{m^2(1-4z+z^2)+q^2(1-x)(1-y)}{m^2(1-z)^2-q^2xy+\mu^2 z} - \frac{m^2(1-4z+z^2)}{m^2(1-z)^2+\mu^2 z}\right]$$

Recalling that $z\to 1$ (and $x,y\to 0$) is the problematic region in feynman parameters for the IR divergence, we can keep the $z\to 1$ divergence in the denominator & remove $x+y$ in the numerator (which would not give singularities):

$$F_1(q^2) \sim \frac{\alpha}{2\pi}\int_0^1 dx\,dy\,dz\,\delta(x+y+z-1)\left[\frac{-2m^2+q^2}{m^2(1-z)^2-q^2xy+\mu^2 z} + \frac{2m^2}{m^2(1-z)^2+\mu^2 z}\right]$$

$$= \frac{\alpha}{2\pi}\int_0^1 dz\int_0^{1-z} dy\left[\frac{-2m^2+q^2}{m^2(1-z)^2-q^2y(1-y-z)+\mu^2} + \frac{2m^2}{m^2(1-z)^2+\mu^2}\right]$$

Set $\mu^2 z=\mu^2$ in denominators, since no singularity.

Change variables: $y=(1-z)\xi$, $w=(1-z)$ $\Rightarrow$ $dw=-dz$, $d\xi=\frac{dy}{1-z}$

$$F_1(q^2) = \frac{\alpha}{2\pi}\int_1^0 -dw\int_0^1 w\,d\xi\left[\frac{-2m^2+q^2}{m^2w^2-q^2w\xi(w-w\xi)+\mu^2} + \frac{2m^2}{m^2w^2+\mu^2}\right]$$

$$= \frac{\alpha}{2\pi}\int_0^1 \frac{d(w^2)}{2}\int_0^1 d\xi\left[\frac{-2m^2+q^2}{w^2(m^2-q^2\xi(1-\xi))+\mu^2} + \frac{2m^2}{m^2w^2+\mu^2}\right]$$

$$= \frac{\alpha}{4\pi}\int_0^1 d\xi\left[\frac{-2m^2+q^2}{m^2-q^2\xi(1-\xi)}\log\left(\frac{m^2-q^2\xi(1-\xi)}{\mu^2}\right) + 2\log\left(\frac{m^2}{\mu^2}\right)\right]$$

Again, the IR divergence is apparent for $\mu^2\to 0$, whether we have finite m_e or q^2.

We obtain

$$F_1(q^2) = 1-\frac{\alpha}{2\pi}f_{IR}(q^2)\log\left(\frac{-q^2 \text{ or } m^2}{\mu^2}\right)+\mathcal{O}(\alpha^2)$$

where $f_{IR} = \int_0^1 \left(\frac{m^2 - q^2/2}{m^2 - q^2 \xi (1-\xi)} \right) d\xi \; - 1$

$f_{IR}(q^2) > 0$ since $q^2 < 0$ and $\max(\xi(1-\xi)) = 1/4$.

We can treat $F_1(q^2)$ as an effective rescaling of the electric charge, since $F_1(q^2)$ is the coefficient of the γ^μ Lorentz structure. Then, $\vec{p} \to \vec{p}\,'$ scattering from a potential is

$$\frac{d\sigma}{d\Omega} \simeq \left(\frac{d\sigma}{d\Omega}\right)_0 \left(1 - \frac{\alpha}{\pi} f_{IR}(q^2) \log\left(-\frac{q^2 \text{ or } m^2}{\mu^2}\right) + \mathcal{O}(\alpha^2)\right)$$

The $\mathcal{O}(\alpha)$ term is an infinitely large negative cross section contribution!

Consider $-q^2 \to \infty$, which isolates the UV behavior of the IR divergence.

$$\int_0^1 d\xi \frac{-q^2/2}{m^2 - q^2 \xi(1-\xi)} \sim \frac{1}{2} \int_0 d\xi \frac{-q^2}{-q^2 \xi + m^2} + \left(\begin{array}{c}\text{equal for} \\ \xi \approx 1\end{array}\right)$$

$$= \log\left(\frac{-q^2}{m^2}\right)$$

So, $F_1(-q^2 \to \infty) = 1 - \frac{\alpha}{2\pi} \log\left(\frac{-q^2}{m^2}\right) \log\left(\frac{-q^2}{\mu^2}\right) + \mathcal{O}(\alpha^2)$

This double log of $(-q^2)$ (Sudakov) is exactly the same structure of the cross section for soft bremsstrahlung:

Compare:

$$\frac{d\sigma}{d\Omega}(p \to p') = \left(\frac{d\sigma}{d\Omega}\right)_0 \left(1 - \frac{\alpha}{\pi} \log\left(\frac{-q^2}{m^2}\right)\left(\log\left(\frac{-q^2}{\mu^2}\right)\right) + \mathcal{O}(\alpha^2)\right)$$

$$\frac{d\sigma}{d\Omega}(p \to p' + \gamma) = \left(\frac{d\sigma}{d\Omega}\right)_0 \left(\frac{\alpha}{\pi} \log\left(\frac{-q^2}{m^2}\right) \log\left(\frac{-q^2}{\mu^2}\right) + \mathcal{O}(\alpha^2)\right)$$

Sum is independent of μ & <u>finite</u>!

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{measured}} \simeq \left(\frac{d\sigma}{d\Omega}\right)_0 \left(1 - \frac{\alpha}{\pi} f_{IR}(q^2) \log\left(\frac{-q^2 \text{ or } m^2}{E^2}\right) + \mathcal{O}(\alpha^2)\right)$$