Lecture 5, November 19, 2020

Thursday, November 19, 2020 2:03 PM

Last time:

Emphasized central role of the Higgs boson in unitarizing EW gauge boson scattering.

One central problem of gauge theories:

Renormalizability of massive gauge bosons

[Distinguish b/t non-Abelian & Abelian]

Higgs, Brout-Engler - etc. → Higgs mechanism that shared SSB gave effective massive gauge bosons in a renormalizeable theory.

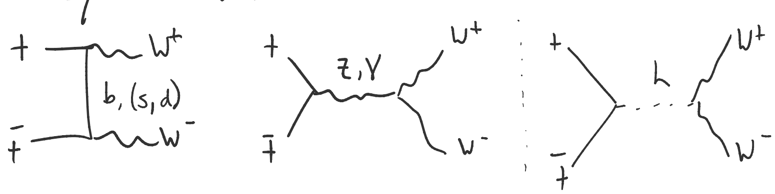
Connection b/t mass & Higgs coupling is a central aspect of Higgs physics currently.

Today: Extend this mass-coupling delicate relation to fermions.

Calculate $f\bar{f} \to ZZ$.

[In general, $f\bar{f} \to W^+W^-$ shows same behavior.

Concretely: $t\bar{t} \to W^+W^-$



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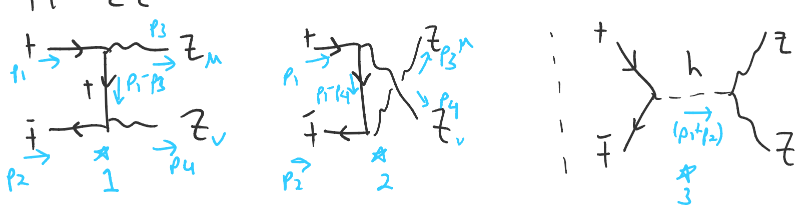
$t\bar{t} \to ZZ$



$$i\mathcal{M}_1 = \bar{v}(p_2) \cdot i\left(g_V \gamma^\nu + g_A \gamma^\nu \gamma^5\right) \cdot \frac{i(\not{p}_1 - \not{p}_3 + m_t)}{(p_1 - p_3)^2 - m_t^2}\, i\left(g_V \gamma^\mu + g_A \gamma^\mu \gamma^5\right) u(p_1) \cdot \epsilon^*_\mu(p_3) \cdot \epsilon^*_\nu(p_4)$$

$$'' \quad \frac{i}{(\not{p}_1 - \not{p}_3) - m_t}$$

$$i\mathcal{M}_2 = \bar{v}(p_2)\, i\left(g_V \gamma^\mu + g_A \gamma^\mu \gamma^5\right) \frac{i(\not{p}_1 - \not{p}_4 + m_t)}{(p_1 - p_4)^2 - m_t^2}\, i\left(g_V \gamma^\nu + g_A \gamma^\nu \gamma^5\right) u(p_1)\; \epsilon^*_\nu(p_4)\, \epsilon^*_\mu(p_3)$$

$$i\mathcal{M}_h = \bar{v}(p_2) \cdot \left(\frac{-i m_t}{v}\right) u(p_1) \cdot \frac{i}{(p_1 + p_2)^2 - m_h^2} \cdot \frac{2 i m_Z^2}{v} \cdot g^{\mu\nu} \cdot \epsilon^*_\mu(p_3)\, \epsilon^*_\nu(p_4)$$

$$i\mathcal{M}_h = \bar{v}(p_2)\left(\frac{-im_t}{v}\right)u(p_1)\cdot\frac{i}{(p_1+p_2)^2-m_h^2}\cdot\frac{2im_Z^2}{v}\cdot g\cdot \epsilon^*_\mu(p_3)\,\epsilon^*_\nu(p_4)$$

Approximate pol. vectors as longitudinal:

$$\epsilon^*_\mu(p_3) \to \frac{p_3^\mu}{m_Z} - \frac{2m_Z}{s}p_4^\mu$$

$$\epsilon^*_\nu(p_4) \to \frac{p_4^\nu}{m_Z} - \frac{2m_Z}{s}p_3^\nu$$

$$i\mathcal{M}_1 = \left(\frac{-i}{t-m_t^2}\right)\cdot\left(\frac{1}{m_Z^2}\right)\bar{v}_2\left(\not{p}_4 - \frac{2m_Z^2}{s}\not{p}_3\right)(g_V+g_A\gamma^5)(\not{p}_1-\not{p}_3+m_t)\left(\not{p}_3-\frac{2m_Z^2}{s}\not{p}_4\right)(g_V+g_A\gamma^5)u(p_1)$$

$$i\mathcal{M}_2 = \left(\frac{-i}{u-m_t^2}\right)\left(\frac{1}{m_Z^2}\right)\bar{v}_2\left(\not{p}_3 - \frac{2m_Z^2}{s}\not{p}_4\right)(g_V+g_A\gamma^5)(\not{p}_1-\not{p}_4+m_t)\left(\not{p}_4-\frac{2m_Z^2}{s}\not{p}_3\right)(g_V+g_A\gamma^5)u(p_1)$$

$$i\mathcal{M}_h = i\frac{2m_t}{v^2}\frac{1}{s-m_h^2}\left(\frac{s}{2}-m_Z^2-\frac{2m_Z^4}{s}-\frac{4m_Z^6}{s^2}\right)\bar{v}(p_2)\,u(p_1)$$

$|\mathcal{M}|^2$

$$\sum_s|\mathcal{M}|^2 = \sum_s(\mathcal{M}_1+\mathcal{M}_2+\mathcal{M}_h)(\mathcal{M}_1+\mathcal{M}_2+\mathcal{M}_h)^\dagger$$

$$\mathcal{M}_1\mathcal{M}_1^\dagger = \left(\frac{1}{t-m_t^2}\right)\left(\frac{1}{t-m_t^2}\right)\frac{1}{m_Z^4}$$

$$\mathrm{Tr}\Big[\left(\not{p}_4-\frac{2m_Z^2}{s}\not{p}_3\right)(g_V+g_A\gamma^5)(\not{p}_1-\not{p}_3+m_t)\left(\not{p}_3-\frac{2m_Z^2}{s}\not{p}_4\right)(g_V+g_A\gamma^5)\cdot(\not{p}_1-m)\left(\not{p}_4-\frac{2m_Z^2}{s}\not{p}_3\right)(g_V+g_A\gamma^5)(\not{p}_1-\not{p}_3+m_t)\left(\not{p}_3-\frac{2m_Z^2}{s}\not{p}_4\right)(g_V+g_A\gamma^5)\cdot(\not{p}_2+m)\Big]$$

$$\sum_s u\bar{u} = \not{p}-m$$

$$\sum_s v\bar{v} = \not{p}+m$$

Aside:

$$\mathcal{M} = \bar{v}_2\,\Gamma\,u_1$$

$$\mathcal{M}^\dagger = \bar{u}_1\,\boxed{\Gamma^\dagger}\,v_2$$

$\Gamma = \gamma_\mu P_L$ ← W-exchange

$\Gamma^\dagger = \gamma_\mu P_L$ or ~~$P_L\gamma_\mu = \gamma_\mu P_R$~~

$$\Gamma = \gamma_\alpha\gamma_\beta$$

$$\Gamma^\dagger = \gamma_\beta\gamma_\alpha$$

$$\mathcal{M} = \bar{v}_2\,\gamma_\alpha(g_V+g_A\gamma^5)\gamma_\beta\gamma_\delta(g_V+g_A\gamma^5)u_1$$

$$\mathcal{M}^\dagger = u_1^\dagger(g_V+g_A\gamma^5)\gamma_\delta^\dagger\gamma_\beta^\dagger(g_V+g_A\gamma^5)\gamma_\alpha^\dagger\,(\bar{v}_2)^\dagger$$

$$\bar{v}_2 = (v_2^\dagger\gamma^0)$$

$$(\bar{v})^\dagger = \gamma^0 \ldots \quad (\gamma^0)^\dagger = \gamma^0$$

$$\bar{v}_2 = (v_2^\dagger \gamma^0)$$

$$(\bar{v}_2)^\dagger = \gamma^0 v_2 \quad \text{since } (\gamma^0)^\dagger = \gamma^0$$

$$\mathcal{M}^\dagger = u_1^\dagger (g_v + g_A \gamma^5) \gamma_\delta^\dagger \gamma_\beta^\dagger (g_v + g_A \gamma^5) \gamma_\alpha^\dagger \gamma^0 v_2$$

$$\gamma_\alpha^\dagger \gamma^0 = \gamma^0 \gamma_\alpha$$

$$= \bar{u}_1 (g_v - g_A \gamma^5) \gamma_\delta \gamma_\beta (g_v - g_A \gamma^5) \gamma_\alpha v_2$$

$$= \bar{u}_1 \gamma_\delta (g_v + g_A \gamma^5) \gamma_\beta \gamma_\alpha (g_v + g_A \gamma^5) v_2$$

Break until 3:10 pm

$$\sum_s \mathcal{M}_1 \mathcal{M}_1^\dagger = \frac{1}{t - m_+^2} \frac{1}{t - m_+^2} \cdot \frac{1}{m_Z^4}$$

$$\mathrm{Tr}\Big[(\not{p}_4 - \frac{2m_Z^2}{s} \not{p}_3)(g_v + g_A\gamma^5)(\not{p}_1 - \not{p}_3 + m)(\not{p}_3 - \frac{2m_Z^2}{s}\not{p}_4)(g_v + g_A\gamma^5)(\not{p}_1 + m)$$
$$(\not{p}_3 - \frac{2m_Z^2}{s}\not{p}_4)(g_v + g_A\gamma^5)(\not{p}_1 - \not{p}_3 + m)(\not{p}_4 - \frac{2m_Z^2}{s}\not{p}_3)(g_v + g_A\gamma^5)(\not{p}_2 - m)\Big]$$

$$\sum_s \mathcal{M}_1 \mathcal{M}_2^\dagger = \frac{1}{t - m_+^2} \frac{1}{u - m_+^2} \cdot \frac{1}{m_Z^4}$$

$$\mathrm{Tr}\Big[(\not{p}_4 - \frac{2m_Z^2}{s} \not{p}_3)(g_v + g_A\gamma^5)(\not{p}_1 - \not{p}_3 + m)(\not{p}_3 - \frac{2m_Z^2}{s}\not{p}_4)(g_v + g_A\gamma^5)(\not{p}_1 + m)$$
$$(\not{p}_4 - \frac{2m_Z^2}{s}\not{p}_3)(g_v + g_A\gamma^5)(\not{p}_1 - \not{p}_4 + m)(\not{p}_3 - \frac{2m_Z^2}{s}\not{p}_4)(g_v + g_A\gamma^5)(\not{p}_2 - m)\Big]$$

$$\sum_s \mathcal{M}_2 \mathcal{M}_1^\dagger = \frac{1}{u - m_+^2} \frac{1}{t - m_+^2} \frac{1}{m_Z^4}$$

$$\mathrm{Tr}\Big[(\not{p}_3 - \frac{2m_Z^2}{s} \not{p}_4)(g_v + g_A\gamma^5)(\not{p}_1 - \not{p}_4 + m)(\not{p}_4 - \frac{2m_Z^2}{s}\not{p}_3)(g_v + g_A\gamma^5)(\not{p}_1 + m)$$
$$(\not{p}_3 - \frac{2m_Z^2}{s}\not{p}_4)(g_v + g_A\gamma^5)(\not{p}_1 - \not{p}_3 + m)(\not{p}_4 - \frac{2m_Z^2}{s}\not{p}_3)(g_v + g_A\gamma^5)(\not{p}_2 - m)\Big]$$

$$\sum_s \mathcal{M}_2 \mathcal{M}_2^\dagger = \frac{1}{u - m_+^2} \frac{1}{u - m_+^2} \frac{1}{m_Z^4}$$

$$\mathrm{Tr}\Big[(\not{p}_3 - \frac{2m_Z^2}{s} \not{p}_4)(g_v + g_A\gamma^5)(\not{p}_1 - \not{p}_4 + m)(\not{p}_4 - \frac{2m_Z^2}{s}\not{p}_3)(g_v + g_A\gamma^5)(\not{p}_1 + m)$$
$$(\not{p}_4 - \frac{2m_Z^2}{s}\not{p}_3)(g_v + g_A\gamma^5)(\not{p}_1 - \not{p}_4 + m)(\not{p}_3 - \frac{2m_Z^2}{s}\not{p}_4)(g_v + g_A\gamma^5)(\not{p}_2 - m)\Big]$$

$$\left(\not{p}_4 - \frac{2m_Z^2}{s}\not{p}_3\right)\left(g_V + g_A\gamma^5\right)\left(\not{p}_1 - \not{p}_4 + m\right)\left(\not{p}_3 - \frac{2m_Z^2}{s}\not{p}_4\right)\left(g_V + g_A\gamma^5\right)\left(\not{p}_2 - m\right)\Big]$$

Again, perform trace + sum.

Expand in powers of $\mathcal{O}(m_Z^n)$:

$$\sum_s |\mathcal{M}|^2_{\substack{\text{gauge}\\ \text{only}}} \supset \frac{1}{m_Z^4} \cdot \frac{-8g_A^2 s}{tu}\left(6g_V^2 m_Z^2 tu + g_A^2\left(m_1^2\left(-s^2 + (t-u)^2\right) + m_Z^2 tu\right)\right) + \mathcal{O}\left(\frac{1}{m_Z^2}\right)$$

The $g_V^2 m_Z^2$ + $g_A^2 m_Z^2$ terms will drop to $\mathcal{O}\left(\frac{1}{m_Z^2}\right)$, need to check other $\mathcal{O}\left(\frac{1}{m_Z^2}\right)$ terms. Will actually not be divergent as $s \to \infty$.

Divergent term: $\frac{1}{m_Z^4} \frac{-8g_A^2 s}{tu}\left(g_A^2\right)\left(m_1^2\right)\left(-s^2 + (t-u)^2\right)$

At $|\mathcal{M}|^2$, among gauge terms, diverges as $\frac{s}{m_Z^2}$, so $\mathcal{M}$ roughly diverges as $\mathcal{O}\left(\frac{\sqrt{s}}{m_Z} = \frac{E}{m_Z}\right)$ slower than what we saw in $WW \to WW$ scattering.

But we have $\frac{g_A^4 m_1^2}{m_Z^4}$ as our prefactor.

Of course, resolution for unitarity violation is the inclusion of Higgs amplitudes + interferences:

$$\sum_s |\mathcal{M}|^2_{\substack{H\text{-gauge}\\ +|H|^2}} \supset \frac{m_1^2}{m_Z^4}\frac{\left(g^4 stu + 8c_W^2 g^2 g_A^2 (s+t-u)(s-t+u)(t+u)\right)}{8c_W^4 tu}$$

Recall Higgs coupling to ZZ:

$$\frac{2m_Z^2}{v} = 2\frac{m_W^2}{v c_W^2} = \frac{2g^2v^2}{4c_W^2 v} = \frac{g^2 v}{2c_W^2}$$

Sum with above:

$$\sum|\mathcal{M}|^2_{tot}\Big|_{\frac{1}{m_Z^4}} = \frac{-8g_A^4 s}{t u\, m_Z^4} m_1^2\left(-s^2+(t-u)^2\right) + \frac{m_1^2}{8c_W^4 t u}\left(g^4 s t u + 8 c_W^2 g^2 g_A^2 (s+t-u)(s-t+u)(t+u)\right)$$

	g_V^Z	g_A^Z
u	$\frac{g}{c_W}\left(\frac{1}{4}-\frac{2}{3}s_W^2\right)$	$\frac{g}{c_W}\left(\frac{-1}{4}\right)$
d	$\frac{g}{c_W}\left(\frac{-1}{4}+\frac{1}{3}s_W^2\right)$	$\frac{g}{c_W}\left(\frac{+1}{4}\right)$
ν	$\frac{g}{c_W}\left(\frac{1}{4}\right)$	$\frac{g}{c_W}\left(\frac{-1}{4}\right)$
e	$\frac{g}{c_W}\left(\frac{-1}{4}+s_W^2\right)$	$\frac{g}{c_W}\left(\frac{+1}{4}\right)$

$$\bar{u}_L \frac{g}{c_W}(\not{Z})(T^3 - s_W^2 Q)u_L$$

P_L

For our case, we only care about g_A^2 or g_A^4, universal for all SM fermions.

$$\sum|\mathcal{M}|^2 = -\frac{s}{t u\, m_Z^4} m_1^2 \cdot \frac{g^4}{c_W^4\cdot 32}\left(-s^2+(t-u)^2\right) + \frac{m_1^2 g^4}{8c_W^4 t u\, m_Z^4}\left(stu + \frac{1}{2}\cdot(s+t-u)(s-t+u)(t+u)\right)$$

$$= \frac{m_1^2 g^4}{8c_W^4 t u\, m_Z^4}\left(stu + \frac{1}{2}(s+t-u)(s-t+u)(t+u) - \frac{s}{4}\left(-s^2+(t-u)^2\right)\right)$$

$$= \frac{g^4 m_1^2 (s+t+u)}{32 c_W^4 t u\, m_Z^4}\left(s^2 - 2(t-u)^2 + s(t+u)\right)$$

$$s+t+u = 2m^2 + 2m^2$$

$s2\ c_w\ \ t\ u\ m_Z$

$$s+t+u = 2m_1^2 + 2m_Z^2$$

No divergence as $s \to \infty$!

Coincidence that was predicted:

Higgs diagram $\sim\ y_f \cdot \frac{m_Z^2}{v}$

(Axial) Gauge diagrams $\sim\ g_A^2 \cdot m_f$ ↙ from fermion propagator

Exactly equal scaling dependence: (in SM)

$$g_A^2 m_f = \frac{g^2}{c_W^2} \cdot y_f v = \frac{g^2 v^2}{c_w^2} \cdot \frac{y_f}{v} = m_Z^2 \underset{\substack{\uparrow \\ \text{Higgs} \\ ff}}{\frac{y_f}{v}} = y_f \underset{\substack{\uparrow \\ \text{Higgs} \\ ZZ}}{\frac{m_Z^2}{v}}$$

Exact coincidence between axial-vector coupling of fermion + Yukawa coupling!

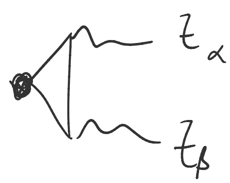
Required by unitarity.

Note: g_V is unconstrained by this argument.

So massive vector boson can have arbitrary vector coupling to fermions regardless of unitarity.

Also, massless vector (like γ + g) are typically purely vector couplings. Would need to consider unbroken phase of EW symmetry to have chiral (axial-vector) coupling to massless gauge bosons.

Adler-Bell-Jackiw U(1) anomaly

$\partial_\mu j^{\mu 5}$ 

$$\partial_\mu j^{\mu 5} \neq 0, \quad \partial_\mu j^{\mu 5} = \frac{g^2}{16\pi^2} Z_{\alpha\gamma} \tilde{Z}_{\beta\delta} \cdot \mathrm{Tr}\left[q_A \{ q_B, q_C \} \right]$$

$$\partial_\mu j^{\mu 5} \neq 0 \,, \quad \partial_\mu j^{\mu 5} = \frac{g^2}{16\pi^2} Z_{\alpha\gamma} \tilde{Z}_{\beta\delta} \cdot \mathrm{Tr}\left[q_A \{ q_B, q_C \} \right]$$

Ganging this current requires have "anomaly-free" sets fermions such that total $\partial_\mu j^{\mu 5} = 0$ when summing over all fermions.

This coincidence $g_A^2 m_f \sim y_f \frac{m_Z^2}{v}$

is also a central aspect to test with Higgs physics.