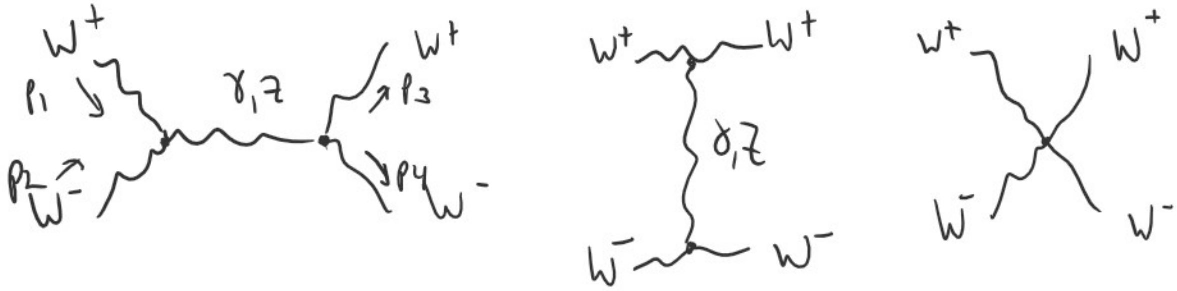
Last time:

WW unitarity violation



$$i\mathcal{M}_{s,\gamma+Z} = -ig^2\left(\frac{s_W^2}{s} + \frac{c_W^2}{s-m_Z^2}\right)\Big((p_1-p_2)_\mu \epsilon_1\cdot\epsilon_2 + 2p_2\cdot\epsilon_1\epsilon_{2\mu} - 2p_1\cdot\epsilon_2\epsilon_{1\mu}\Big)$$
$$\Big((p_4-p_3)^\mu \epsilon_3^*\cdot\epsilon_4^* + 2p_3\cdot\epsilon_4^*\epsilon_3^{*\mu} - 2p_4\cdot\epsilon_3^*\epsilon_4^{*\mu}\Big)$$

$$\Bigg[\ Z\text{-propagator: } \underbrace{\frac{-i}{s-m_Z^2}}_{\text{only remaining contrib.}}\left(g^{\mu\nu} - \frac{(p_1+p_2)^\mu (p_1+p_2)^\nu}{m_Z^2}\right)$$

cancels when contracting the 3pt. vertices

$$i\mathcal{M}_{t,\gamma+Z} = ig^2\left(\frac{s_W^2}{t} + \frac{c_W^2}{t-m_Z^2}\right)\Big(-(p_1+p_3)_\mu \epsilon_1\cdot\epsilon_3^* + 2p_1\cdot\epsilon_3^*\epsilon_{1\mu} + 2p_3\cdot\epsilon_1\epsilon_{3\mu}^*\Big)$$
$$\Big((p_2+p_4)^\mu \epsilon_2\cdot\epsilon_4^* - 2p_4\cdot\epsilon_2\epsilon_4^{*\mu} - 2p_2\cdot\epsilon_4^*\epsilon_2^\mu\Big)$$

$$i\mathcal{M}_4 = ig^2\Big(2\epsilon_1\cdot\epsilon_4^*\epsilon_2\cdot\epsilon_3^* - \epsilon_1\cdot\epsilon_3^*\epsilon_2\cdot\epsilon_4^* - \epsilon_1\cdot\epsilon_2\,\epsilon_3^*\cdot\epsilon_4^*\Big)$$

Analyze longitudinal gauge bosons:

$\frac{m_W^2}{s}$ exp. in CoM frame

Approximate:

$$\epsilon_{1L}^\mu = \frac{p_1^\mu}{m_W} - \frac{2m_W}{s}p_2^\mu = \frac{1}{m_W}\left(p_1^\mu - \frac{2m_W^2}{s}p_2^\mu\right)$$

in the CoM frame.

$$\left[\begin{array}{l}\epsilon_i\cdot\epsilon_j = -\delta_{ij} \leftarrow \\ \epsilon\cdot p = 0\end{array}\right.$$

$$\epsilon_{2L}^\mu = \frac{p_2^\mu}{m_W} - 2m_W\, p_1^\mu$$

$L\ \epsilon \cdot p = 0$

Massless: 2 transverse

Massive: 2 trans + 1 long.

$$\epsilon_{2L}^{\mu} = \frac{p_2^{\mu}}{m_W} - \frac{2m_W}{s} p_1^{\mu}$$

frame.

Equivalently for ϵ_3, ϵ_4 + expand in terms $\mathcal{O}(m_W^n)$

$$i\mathcal{M}_{\text{tot, gauge}} = \frac{-ig^2}{4m_W^4}\left(s^2 + 2st + t^2 - su - tu - 2u^2\right)$$

$$+ \frac{ig^2}{4sm_W^2}\left(7s^2 - 8t(t+u) - s(t+(u))\right)$$

$$+ \mathcal{O}\left(m_W^0, \frac{m_W^2}{s}\ldots\right)$$

$[s]^2 = \left((p_1+p_2)^2\right)^2 = (E_{CM})^4$

Naïvely, scales as $\mathcal{O}\left(\frac{E^4}{m_W^4}\right)$ in first line.

* Leads to unitarity violation, since divergence in $\mathcal{M}$ leads to σ- probability becomes larger than 1.

For on-shell W's, one constraint on (s, t, u):

$$s + t + u = 4m_W^2.$$

Reshuffle first term: no quadratic dependence at all

$$[s^2 + 2st + t^2 - su - tu - 2u^2$$

$$= s(s+2t) + t(t-u) - u(s+2u)$$

$$= s(t + 4m_W^2 - u) + t(t-u) - u(u + 4m_W^2 - t)$$

$$= st + 4m_W^2 s - us + t^2 \cancel{-ut} - u^2 - 4m_W^2 u + \cancel{tu}$$

$$= (s+t)(s+t) - (s+t+u)u - u^2$$

$$= (4m_W^2 - u)(4m_W^2 - u) - 4m_W^2 u - u^2$$

$$= 16m_W^4 - 8m_W^2 u + \cancel{u^2} - 4m_W^2 u - \cancel{u^2}$$

$$= 16m_W^4 - 12m_W^2 u$$

$$= 4m_W^2(4m_W^2 - 3u)$$ ★ Gauge structure ensures naive $\mathcal{O}\left(\frac{E^4}{m_W^4}\right)$ scaling vanishes

$$i\mathcal{M}_{\text{tot, gauge}} = \frac{-ig^2}{4m_W^4} \cdot 4m_W^2(4m_W^2 - 3u)$$

$$+ \frac{ig^2}{4s\,m_W^2}\left(7s^2 - 8t(t+u) - s(t+6u)\right) + \mathcal{O}\left(m_W^0, \frac{m_W^2}{s}, \dots\right)$$

But also cannot have $\mathcal{O}\left(\frac{E^2}{m_W^2}\right)$ scaling.

Massage terms in green:

$$\frac{3ig^2 u}{m_W^2} + \frac{ig^2}{m_W^2}\frac{1}{4s}\left(7s^2 - 8t(t+u) - s(t+6u)\right)$$

$$= \frac{ig^2}{m_W^2}\left(\frac{1}{4s}\right)\left(12su + 7s^2 - 8t(4m_W^2 - s) - st - 6us\right)$$

$$= \frac{ig^2}{m_W^2}\left(\frac{1}{4s}\right)\left(6su + 7s^2 - 32tm_W^2 + 7st\right)$$

$$= \frac{ig^2}{m_W^2}\left(\frac{1}{4s}\right)\left(7s(s+t+u) - su - 32tm_W^2\right)$$

$$= \frac{ig^2}{m_W^2}\left(\frac{1}{4s}\right)\left(7s \cdot 4m_W^2 - su - 32tm_W^2\right)$$

$$= \frac{ig^2}{m_W^2}\left(\underbrace{7m_W^2}_{\text{drop}} - \underbrace{\frac{u}{4}}_{\text{grows as } \mathcal{O}\left(\frac{E^2}{m_W^2}\right)} - \underbrace{\frac{8t}{s}m_W^2}_{\text{drop}}\right)$$

$\mathcal{O}(m_W^0)$

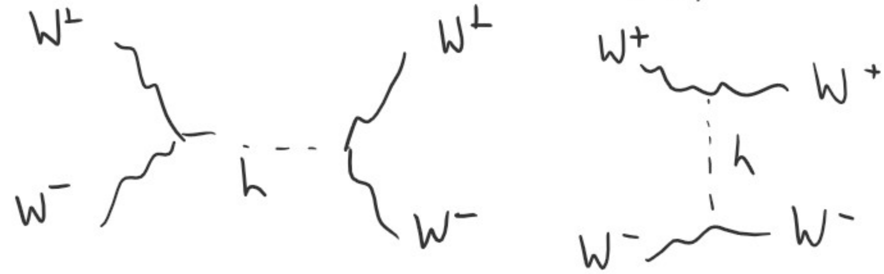
So $-\frac{ig^2}{4m_W^2}u$ is leading term in $i\mathcal{M}_{\text{tot}}$.

So $-\frac{ig^2}{4m_W^2}u$ is leading term in $i\mathcal{M}_{tot}$.

Unitarity is violated if we only consider gauge diagrams!

How do we save unitarity?

Need new particle: Higgs mechanism predicts the Higgs diagrams (s + t -channel).



$$i\mathcal{M}_{s,h} = -i\,\epsilon_1\cdot\epsilon_2\,\epsilon_3^*\cdot\epsilon_4^*\,\frac{g^2 m_W^2}{s-m_h^2}$$

$$i\mathcal{M}_{t,h} = -i\,\epsilon_1\cdot\epsilon_3^*\,\epsilon_2\cdot\epsilon_4^*\,\frac{g^2 m_W^2}{t-m_h^2}$$

Apply the longitudinal approx., expand in $\mathcal{O}(m_W^n; m_h^{n_2})$:

$$i\mathcal{M}_{tot,\,Higgs} = \underbrace{\frac{-ig^2}{4m_W^2}(s+t)}_{} - \frac{ig^2 m_h^2}{2m_W^2} + \mathcal{O}\left(m_W^0, m_h^0, \frac{m_W^2}{s}, \frac{m_h^2}{s}\right)$$

The Higgs diagrams resolve unitarity violation in gauge diagrams:

$$i\mathcal{M}_{tot} = \frac{-ig^2}{4m_W^2}(u) - \frac{ig^2}{4m_W^2}(s+t) - \frac{ig^2 m_h^2}{2m_W^2} + \mathcal{O}\left(m_W^0, m_h^0 \dots\right)$$

$$= \underbrace{-ig^2}_{} - \frac{ig^2 m_h^2}{2m_W^2} + \dots$$

No $\mathcal{O}(E^2)$ unitarity violation at all

$$\underbrace{}_{2m_W^2}$$

No $\mathcal{O}\left(\frac{E^2}{m_W^2}\right)$ unitarity violation at all.

Return @ 3:20pm.

No $\mathcal{O}\left(\frac{E^2}{m_W^2}\right) \Rightarrow$ becomes $\frac{-ig^2 m_h^2}{2m_W^2}\left(K_V^2\right) + \frac{-ig^2 m_{h_2}^2}{2m_W^2}\left((1-K_V^2)\right)$

For $m_h^2 \gg m_W^2$, also have unitarity violation

Leads to "no-lose" theorem, which guaranteed the existence of Higgs boson for perturbative unitarity restoration or some other new physics that resolved unitarity problem.

Essentially, $m_h < 800-1000$ GeV by this argument.

$$\frac{-ig^2 m_h^2}{2m_W^2} = \frac{-ig^2(2\lambda v^2)}{\left(\frac{g^2 v^2}{2}\right)} = \underbrace{-i4\lambda}$$

So, perturbative unitarity violation by making $m_h \gg m_W$ is equivalent to making $\lambda \gg 1$.

Comment 1: Precise bound comes from studying partial wave unitarity. Schwartz: 24.1.5, +29.2.

Comment 2: All of this discussion does not care about matter content.

Comment 3: Simplify the punchline to:

Only choose params. in self-consistent way.

Trying to study SM deviations: Large deviations imply

Trying to study SM deviations: Large deviations imply new physics to self-consistently predict the deviations you want to measure.

Ex. In Higgs physics, introduce coupling deviation κ_V, rescales Higgs-vector-vector strength. ($\kappa_V = 1$ is SM limit)

$$i\mathcal{M}_{tot} = \frac{-ig^2}{4m_W^2}(s+t)\cdot\kappa_V^2 - \frac{ig^2 m_h^2}{2m_W^2}\cdot\kappa_V^2 - \frac{ig\,u}{4m_W^2}$$

$$= -ig^2\kappa_V^2 \underbrace{- \frac{ig^2 u}{4m_W^2}(1-\kappa_V^2)}_{\text{reintroduce unitarity violation}} - \frac{ig^2 m_h^2}{2m_W^2}\kappa_V^2$$

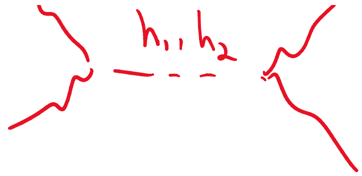
The NP responsible for this unitarity violation is absent in the $\mathcal{L}$ $\Rightarrow$ lost predictivity for all energy scales by not being self-consistent.

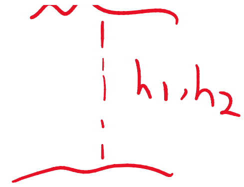
Recognize that this is, at best, an effective description for the physics.

Even renormalizable theories can have effective ranges of validity!

$$\kappa_V \rightarrow h_1\,\kappa_V\,\frac{m_W^2}{v}\,W^+_\mu W^{-\mu} + h_2\,(1-\kappa_V)\,\frac{m_W^2}{v}\,W^+_\mu W^{-\mu}$$

2HDM UV completion.





$$i\mathcal{M}_{h,s} = \left(\frac{\kappa_V^2}{s-m_{h_1}^2} + \cancel{\frac{(1-\kappa_V)^2}{s-m_{h_2}^2}}\right) + \text{same for } t\text{-}\ldots$$

$$\langle h_1 \rangle = v\sqrt{\kappa_V}$$

$$\langle h_2 \rangle = v\sqrt{(1-\kappa_V)}$$

$$\langle h_1 \rangle^2 + \langle h_2 \rangle^2 = v^2$$

resolves inconsistency b/t W mass + scalar-W-W couplings

Similar story:

g_A + m_f in $f\bar{f} \to ZZ$.