Lecture 3, November 11, 2020

Wednesday, November 11, 2020 2:08 PM

Reminder: HW 1 due tomorrow

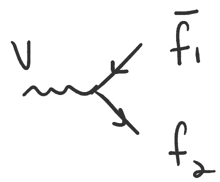
Volunteers? (PhD students, please)

Alexey - prob. 1

? - prob. 2

Last time: SM at tree-level

Concluded with calculation of W + Z branching fractions.



$\Gamma(V \to \bar{f}_1 f_2)$ decay.

W+Z branching fractions pheno.

Ex. What are the dominant W decays + why?

What are the dominant Z decays + why?

Ans: $W \to f_1 \bar{f}_2$, $Z \to f\bar{f}$

diff. f's.

$m_Z \sim 91.2\,\text{GeV}$ $m_b \sim 4.2\,\text{GeV}$

$m_W \sim 80.4\,\text{GeV}$

$$\Gamma(V \to f_1 \bar{f}_2) = \frac{1}{12\pi}\frac{1}{m_V^3}\sqrt{\lambda(m_V^2, m_1^2, m_2^2)} \cdot N_c$$
$$\left(\frac{1}{2}(g_V^2 + g_A^2)\left(2m_V^2 - m_1^2 - m_2^2 - \frac{(m_1^2 - m_2^2)^2}{m_V^2}\right)\right.$$
$$\left. + 3(g_V^2 - g_A^2)\, m_1 m_2\right)$$

In broken phase, 3 flavors of ν, e^-, u, d.

Flavors: gauge coupling to different flavors are exactly the same, by definition.

the same, by definition.

Only exception: W interactions involve CKM entries.

In the SM, can generally approx. CKM as $\mathbb{1}_{3\times 3}$.

Hence, mass of fermion & T_3 rep./ EW charge are the only distinguishing features between diff. partial widths.

$$\Gamma(Z\to e^+e^-) = \Gamma(Z\to \mu^+\mu^-) = \Gamma(Z\to\tau^+\tau^-) \text{ if}$$

we set $m_e = m_\mu = m_\tau$. Very good in the SM since all $m_{\ell^+} \ll m_Z$.

$$\Gamma(Z\to \nu_1\bar\nu_1) = \Gamma(Z\to\nu_2\bar\nu_2) = \Gamma(Z\to\nu_3\bar\nu_3)$$

since $m_\nu \approx 0$.

$$\Gamma(Z\to u\bar u) = \Gamma(c\bar c) \boxed{\neq \Gamma(t\bar t) \text{ if } m_u = m_c \neq m_t.}$$

$m_t > \frac{m_Z}{2}$.

$$\Gamma(Z\to d\bar d) = \Gamma(s\bar s) = \Gamma(b\bar b)$$

if $m_d \approx m_s \approx m_b$ or rather, $m_{d_i} \ll m_Z$.

$m_b \approx 4.2$ GeV (pole)

$m_s \approx 100$ MeV ($\overline{MS}$)

$m_d \approx 4$-6 MeV ($\overline{MS}$)

$m_t \approx 173.2$ GeV (pole)

$m_c \approx 1.2$ GeV ($\overline{MS}$)

$m_u \approx 2$-5 MeV ($\overline{MS}$)

$m_\tau \approx 1.777$ GeV

$m_\mu \approx 105.1$ MeV

$m_e \approx 511$ keV

$$\Gamma(V\to f_1\bar f_2) = \frac{1}{12\pi}\frac{1}{m_V^3}\sqrt{\lambda(m_V^2, m_1^2, m_2^2)}\cdot N_c \left(\frac{1}{2}(g_V^2+g_A^2)\left(2m_V^2 - m_1^2 - m_2^2 - \frac{(m_1^2-m_2^2)^2}{m_V^2}\right) + 3(g_V^2 - g_A^2)\, m_1 m_2\right)$$

$$+3(g_V^2-g_A^2)\,m_1 m_2\Big)$$

$$\Gamma(V\to f_1\bar f_2)\simeq \frac{1}{12\pi}\frac{1}{\cancel{m_V^3}}m_V\left(\cancel{\tfrac{1}{2}}(g_V^2+g_A^2)\cancel{2}\cancel{m_V^2}\right)N_c\,\cancel{m_V^2}$$

$m_V \gg m_1, m_2$

$$\approx \frac{1}{12\pi}\,m_V N_c\left(g_V^2+g_A^2\right)$$

The fact that the first 5 flavors of quarks can usually be approx. as massless + negligible compared to EW $m_W + m_Z$ is very helpful at simplifying expressions.

What are the charges of u, d, e, ν?

It is useful to express the Z- & W-mediated forces as the currents of fermion bilinears.

$$\overline{\Psi i\not{D}\Psi} = \bar\Psi i\not\partial\Psi + \left(\bar\Psi\frac{g}{\sqrt2}(\not{W}^+T^+ + \not{W}^-T^-)\Psi\right)$$
$$+\left(\bar\Psi\frac{g}{c_W}\not{Z}(T^3-\underline{s_W^2 Q})\Psi\right)+\left(\bar\Psi e\not{A}(T^3+Y)\Psi\right)$$

Now, EOM of gauge field, $\dfrac{\delta\mathcal{L}}{\delta A_\mu}=J_\mu$.

$$\mathcal{L}\supset g\left(W^+_\mu J^{\mu+}_W + W^-_\mu J^{\mu-}_W + Z^0_\mu J^\mu_Z\right)+eA_\mu J^\mu_{EM}$$

$$J^{\mu+}_W=\frac{1}{\sqrt2}\left(\bar\nu_L\gamma^\mu e_L+\bar u_L\gamma^\mu d_L\right),\quad J^{\mu-}_W = h.c.(J^{\mu+}_W)$$

$$J^\mu_Z=\frac{1}{c_W}\Big(\bar\nu_L\gamma^\mu\tfrac{1}{2}\nu_L+\bar e_L\gamma^\mu\left(-\tfrac12+s_W^2\right)e_L+\bar e_R\gamma^\mu s_W^2 e_R$$
$$+\bar u_L\gamma^\mu\left(\tfrac12 - 2\ldots\right)\ldots$$

$$\ldots \frac{1}{c_W}\left(\ldots s_W^2\right) e_L + \bar{e}_R \gamma^\mu\left(s_W^2\right) e_R$$

$$+ \bar{u}_L \gamma^\mu \left(\frac{1}{2} - \frac{2}{3} s_W^2\right) u_L + \bar{u}_R \gamma^\mu \left(\frac{-2}{3} s_W^2\right) u_R$$

$$+ \bar{d}_L \gamma^\mu \left(\frac{-1}{2} + \frac{1}{3} s_W^2\right) d_L + \bar{d}_R \gamma^\mu \left(\frac{1}{3} s_W^2\right) d_R \Big)$$

$$J^\mu_{EM} = \bar{e}\, \gamma^\mu (-1) e + \bar{u} \gamma^\mu \left(\frac{2}{3}\right) u + \bar{d} \gamma^\mu \left(\frac{-1}{3}\right) d$$

Detail: Γ written with $g_V + g_A$ coefficients.
Translate $J_{W+/-}$ + J_Z into $g_V + g_A$ notation.

$$J^\mu_{W+} = \frac{1}{\sqrt{2}} \left(\bar{\nu} \left(\frac{\gamma^\mu}{2} - \frac{\gamma^\mu \gamma^5}{2}\right) e + \bar{u} \left(\frac{\gamma^\mu}{2} - \frac{\gamma^\mu\gamma^5}{2}\right) d \right)$$

$$J^\mu_Z = \frac{1}{c_W} \Big(\bar{\nu} \gamma^\mu \left(\frac{1}{4} - \frac{\gamma^5}{4}\right) \nu + \bar{e}\, \gamma^\mu \left(\frac{-1}{4} + s_W^2 + \frac{1}{4}\gamma^5\right) e$$

$$+ \bar{u} \gamma^\mu \left(\frac{1}{4} - \frac{2}{3} s_W^2 - \frac{1}{4}\gamma^5\right) u + \bar{d}\, \gamma^\mu \left(\frac{-1}{4} + \frac{1}{3} s_W^2 + \frac{1}{4}\gamma^5\right) d \Big)$$

Now, can read off $g_V + g_A$ couplings.

$$\Gamma(Z \to f\bar{f}) \simeq \frac{1}{12\pi} m_Z N_c \left(g_V^2 + g_A^2\right)$$

$$\mathcal{L} \supset \bar{\Psi} \left(g_V \gamma^\mu + g_A \gamma^\mu \gamma^5\right) Z_\mu \Psi$$

In the SM, $s_W^2 \simeq 0.22$ or 0.225

g_V^2 for $e \simeq 0$. accidentally.

Break until 3:18

Numerics: $Br(Z \to \text{hadrons, e.g. } q\bar{q}) \simeq 70\%$

$Br(Z \to \nu\bar{\nu}) \simeq 20\%$

$Br(Z \to \ell^+\ell^-) \simeq 10\%$.

W decays only distinguish based on multiplicity:

Kinematics only allow:

$$W^- \to \underbrace{\bar{u}\,d,\ \bar{c}\,s}, \ e\bar{\nu}$$

$$q\bar{q}' = \underset{\text{flavors}}{2} \times \underset{N_c}{3}$$

$$l\nu = \underset{\text{flavors}}{3} \times \underset{N_c}{1}$$

At tree-level, we have a factor of 9 multiplicity with equal couplings (+ masses neglected).

Basic unit of $Br(W \to ff')$ @ tree-level is 11%. Easy to partition.

$Br(W \to e\nu) = 11\%$, $Br(W \to \mu\nu) = 11\%$, $Br(\tau\nu) = 11\%$

$Br(W \to u\bar{d}) = 33\%$, $Br(W \to c\bar{s}) = 33\%$.

What about Cabibbo correction?

Must be self-consistent in treatment of mixing angle.

$$\Gamma(W \to u\bar{d})\Big|_{CKM=\mathbb{1}} + \Gamma(W \to c\bar{s})\Big|_{CKM=\mathbb{1}}$$

$$\quad\ \ 1 \qquad\qquad\qquad\qquad 1$$

$$= \left(\Gamma(W \to u\bar{d}) + \Gamma(W \to u\bar{s}) + \Gamma(W \to c\bar{d}) + \Gamma(W \to c\bar{s})\right)\Big|_{CKM = \begin{pmatrix} c_\theta & s_\theta & 0 \\ -s_\theta & c_\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}}$$

$$\downarrow \qquad \downarrow \qquad \downarrow \qquad \downarrow$$

$$c_\theta^2 \qquad s_\theta^2 \qquad s_\theta^2 \qquad c_\theta^2$$

Have finished tree-level for W + Z.

Next, could do Higgs @ tree-level.

But, this will be incomplete.

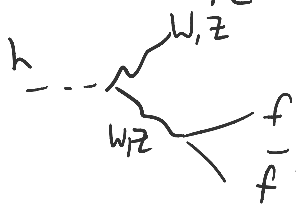
Higgs: Tree-level fails because of

1.) Higgs Yukawas are small ($b\bar{b}, c\bar{c}$, etc.)

$$y_b \sim \frac{1}{60} \quad (\text{at } \mu = m_H)$$

2.) Higgs mass is between $m_V < m_H < 2m_V$

is not 2-body



phase space suppression $\frac{1}{8\pi}$ compared to tree level.

3.) One-loop have $\mathcal{O}(1)$ couplings.

Go back to J_Z^μ:

$$J_Z^\mu = \frac{1}{c_W}\Big(\bar{\nu}\gamma^\mu\left(\frac{1}{4} - \frac{\gamma^5}{4}\right)\nu + \bar{e}\gamma^\mu\left(\frac{-1}{4} + s_W^2 + \frac{1}{4}\gamma^5\right)e$$
$$+ \bar{u}\gamma^\mu\left(\frac{1}{4} - \frac{2}{3}s_W^2 - \frac{1}{4}\gamma^5\right)u + \bar{d}\gamma^\mu\left(\frac{-1}{4} + \frac{1}{3}s_W^2 + \frac{1}{4}\gamma^5\right)d\Big)$$

The axial-vector coupling is <u>prescribed</u>.

It's straightforward to have new vector couplings of matter fields to heavy (e.g. not massless) gauge bosons.

$$\mathcal{L} \supset g_X Z'_\mu J^\mu_{Z'}$$

In limit that $g_X \to 0$, we get SM Lagrangian.

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In $J^\mu_{Z'}$, would have to distinguish between γ^μ + $\gamma^\mu \gamma^5$ coupling (or charges).

Important unitarity argument about axial vector couplings for fermions.

In fact, there is also a critical unitarity argument about weak, massive gauge boson scattering; both kinds can be studied at tree-level.

Unitarity + EWSB.

Prior to the Higgs discovery, we had a "no-lose" theorem that the LHC (or any sufficiently high energy machine) would discover a Higgs or some other phenomena.

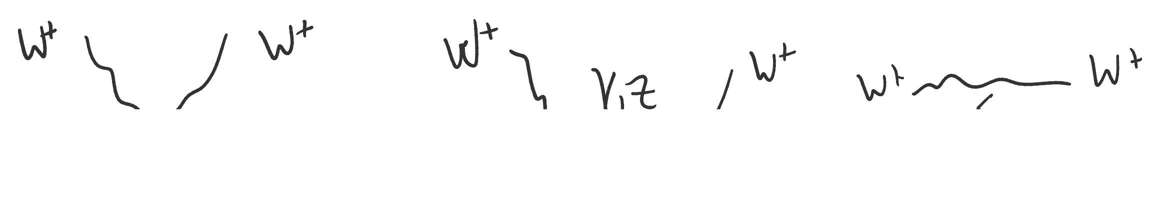
Lee, Quigg, Thacker (1977).

Calculate 2-to-2 scattering of massive EW gauge bosons.

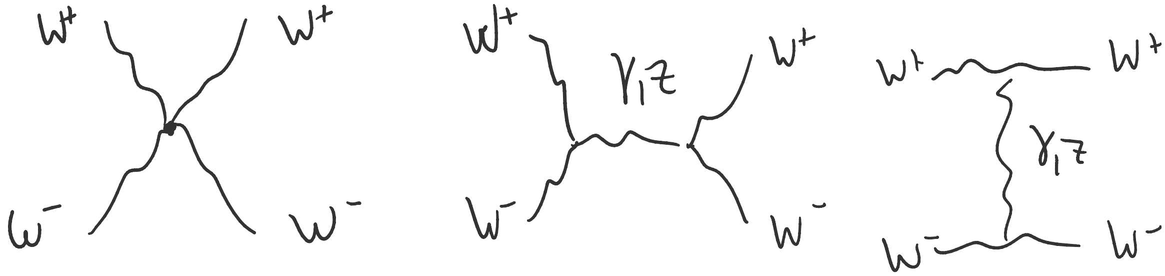
Recall that massless vectors have 2 dofs (2 transverse pols.), while massive gauge bosons have 3 dofs (2 trans. + 1 long.).

Next time: Analyze

$W^+W^- \to W^+W^-$.



pure gauge



w/ Higgs