Lecture 2, November 5, 2020

Thursday, November 5, 2020 2:04 PM

Last time: Motivation for QFT3 & SM pheno.
EWSB & gauge boson masses.

Today: The SM at tree level

Last time, we reviewed EW mixing angle, defined by basis change from gauge basis of W^3, B to mass basis of Z_μ & A_μ.

We have a similar story w/ SM charged fermions.

$$\mathcal{L}_{Yuk} = -y_u \bar{Q}_L \tilde{H} u_R - y_d \bar{Q}_L H d_R - y_e \bar{L}_L H e_R + h.c.$$

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix}, \quad Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \quad L_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$$

$$\tilde{H} = i\sigma_2 H^* = \frac{1}{\sqrt{2}}\begin{pmatrix} v+h \\ 0 \end{pmatrix}$$

Focus only on vev,

$$\mathcal{L}_{Yuk}\big|_v = -y_u \frac{v}{\sqrt{2}} \bar{u}_L u_R - y_d \frac{v}{\sqrt{2}} \bar{d}_L d_R - y_e \frac{v}{\sqrt{2}} \bar{e}_L e_R + h.c.$$

Yukawa matrices are 3×3 complex.

h.c. terms: $\left[(\bar{u}_L u_R)^\dagger = ((u_L)^\dagger \gamma^0 u_R)^\dagger \right.$

$$= \left((P_L u)^\dagger \gamma^0 P_R u \right)^\dagger$$

$$\bar{u}_L = \overline{(u_L)} \checkmark \qquad \cancel{(\bar{u})_L}$$

$$= \left(\left(\left(\frac{1-\gamma_5}{2}\right) u \right)^\dagger \gamma^0 \left(\frac{1+\gamma_5}{2}\right) u \right)^\dagger$$

$$= \left(u^\dagger \left(\frac{1-\gamma_5}{2}\right) \gamma^0 \left(\frac{1+\gamma_5}{2}\right) u \right)^\dagger$$

$$\left[u^\dagger \left(\frac{1+\gamma_5}{2}\right)^\dagger \left(\frac{1+\gamma_5}{2}\right) u\right)$$

$$= u^\dagger \left(\frac{1+\gamma_5}{2}\right) (\underbrace{\gamma^0)^\dagger}_{= \gamma_0} \left(\frac{1-\gamma_5}{2}\right) u$$

$$= u^\dagger \gamma_0 \left(\frac{1-\gamma_5}{2}\right) u$$

$$= \bar{u} P_L u = \bar{u}\, u_L = \overline{u_R}\, u_L \Big]$$

Write h.c. explicity + put in flavor indices.

$$\mathcal{L}_{Yuk}\big|_v = -y_u^{ij} \frac{v}{\sqrt{2}} \bar{u}_L^i u_R^j - y_d^{ij} \frac{v}{\sqrt{2}} \bar{d}_L^i d_R^j - y_e^{ij} \frac{v}{\sqrt{2}} \bar{e}_L^i e_R^j$$

$$- (y_u^{ji})^* \frac{v}{\sqrt{2}} \bar{u}_R^j u_L^i - (y_d^{ji})^* \frac{v}{\sqrt{2}} \bar{d}_R^j d_L^i - (y_e^{ji})^* \frac{v}{\sqrt{2}} \bar{e}_R^j e_R^i$$

How to proceed?

Notice that SM Lagrangian w/ no Yukawa terms has $U(3)^5$ flavor symmetry, since each chiral set of fermions can be rotated into each other by $U(3)$ symmetry.

$$\bar{Q}_L i\not{\partial} Q_L = \bar{Q}_L U_Q^\dagger U_Q\, i\not{\partial}\, U_Q^\dagger U_Q Q_L$$

$$= \bar{\tilde{Q}}_L \underbrace{U_Q U_Q^\dagger}_{= \mathbb{1}} i\not{\partial}\, \tilde{Q}_L \,, \text{ for } \tilde{Q}_L \equiv U_Q Q_L$$

Decompose $U(3) = U(1) \times SU(3)$.

Use $SU(3)$ freedom to diagonalize Yukawa couplings.

$$\mathcal{L}_{Yuk}\big|_v = -\frac{v}{\sqrt{2}} \bar{u}_L V_{u_L}^\dagger V_{u_L} y_u V_{u_R}^\dagger V_{u_R} u_R - \frac{v}{\sqrt{2}} \bar{d}_L V_{d_L}^\dagger V_{d_L} y_d V_{d_R}^\dagger V_{d_R} d_R$$

$$- \frac{v}{\sqrt{2}} \bar{e}_L V_{e_L}^\dagger V_{e_L} y_e V_{e_R}^\dagger V_{e_R} e_R + h.c.$$

Now, choose V's s.t. $V_{u_L} y_u V^\dagger_{u_R}$ = diag ...

Now, choose V's s.t. $V_{u_L} y_u V_{u_R}^\dagger = y_u^{diag}$ + etc.

Redefine $u_R^m = V_{u_R} u_R$, $u_L^m = V_{u_L} u_L$, etc.

$$= -\frac{v}{\sqrt{2}} y_u^{diag} \bar{u}_L^m u_R^m - \frac{v}{\sqrt{2}} y_d^{diag} \bar{d}_L^m d_R^m$$
$$- \frac{v}{\sqrt{2}} y_e^{diag} \bar{e}_L^m e_R^m \quad + h.c.$$

[The $U(1)^5$ is a bit complicated to count, will do it next time.] ☆ For the moment, assume diag entries are all real.

$$= -\frac{v}{\sqrt{2}} y_u^{diag} \left(\bar{u}_L^m u_R^m + \bar{u}_R^m u_L^m\right] + \dots (d, e)$$

$$= -\frac{v}{\sqrt{2}} y_u^{diag} \left[\bar{u}^m (P_R + P_L) u^m\right] + \dots$$

$$= -\frac{v}{\sqrt{2}} y_u^{diag} \bar{u} u$$

In SM, all γ_5 couplings vanish.

Now, [WLOG, order by increasing size]

Identify $[m_u, m_c, m_t] \equiv \left[\frac{v}{\sqrt{2}} y_{u\,11}^{diag}, \frac{v}{\sqrt{2}} y_{u\,22}^{diag}, \frac{v}{\sqrt{2}} y_{u\,33}^{diag}\right]$

+ same for down-type quarks + leptons.

Remark: The original y_u, y_d, y_e were always arbitrary! Always have enough flavor symmetry to choose diagonal mass basis.

Note that we needed 6 V's to get all Yukawa matrices diag'l. So, the $\bar{Q}_L i \not{D} Q_L$ gauge interactions will now be <u>non-diagonal</u>.

Explicitly,

$$\bar{Q}_L \, i\cancel{D} \, Q_L = \bar{u}_L \, i\cancel{\partial} \, u_L + \bar{u}_L \, g_s \cancel{G}^a T^a \, u_L$$
$$+ \bar{u}_L \, \frac{g}{c_w} \cancel{Z} \left(T^3 - s_w^2 Q\right) u_L + \bar{u}_L \, e \cancel{A} \, Q \, u_L$$
$$+ (\text{same for } d_L)$$
$$+ \bar{u}_L \frac{g}{\sqrt{2}} \cancel{W}^+ d_L + \bar{d}_L \frac{g}{\sqrt{2}} \cancel{W}^- u_L$$

Adopting mass basis,
first rows:

$$\bar{u}_L V_{u_L}^\dagger V_{u_L} \, i\cancel{\partial} \, V_{u_L}^\dagger V_{u_L} u_L = \bar{u}_L^m V_{u_L} V_{u_L}^\dagger \, i\cancel{\partial} \, u_L^m$$
$$= \bar{u}_L^m \, i\cancel{\partial} \, u_L^m +$$

So all neutral current interactions remain flavor conserving. same for $\cancel{G}^a T^a$, $\cancel{Z}$, $\cancel{A}$

On the other hand, the $W^{+/-}$ interactions are flavor-violating.

$$\bar{u}_L V_{u_L}^\dagger V_{u_L} \frac{g}{\sqrt{2}} \cancel{W}^+ V_{d_L}^\dagger V_{d_L} d_L + \text{h.c.}$$

$$= \bar{u}_L^m V_{u_L} V_{d_L}^\dagger \frac{g}{\sqrt{2}} \cancel{W}^+ d_L^m + \text{h.c.}$$

This matrix $V_{u_L} V_{d_L}^\dagger \equiv V_{CKM}$ is the only source of flavor violation in quark interactions.

[Of course, massive neutrinos have equivalent PMNS matrix.]

So Feynman rules for charged current exchange must include V_{CKM} factor.

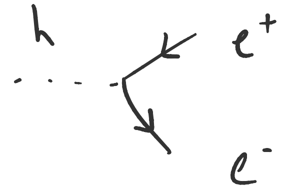
Higgs Yukawa + gauge interactions.

Easiest way to derive Higgs couplings is a pseudo-spurion analysis, by replacing $v \to v+h$.

Yukawa couplings:

$$\mathcal{L}_{Yuk}\Big|_h = -\frac{h}{\sqrt{2}} y_u^{diag} \bar{u}u - \frac{h}{\sqrt{2}} y_d^{diag} \bar{d}d - \frac{h}{\sqrt{2}} y_e^{diag} \bar{e}e .$$

Ex.



$$\frac{-iy_e''}{\sqrt{2}} = -i\frac{m_e}{v} \quad \text{or} \quad \frac{-im_f}{v} \text{ for fermion } f.$$

Note: $m_e = \frac{y_e'' v}{\sqrt{2}} \Rightarrow \frac{m_e}{v} = \frac{y_e''}{\sqrt{2}}$

Break: 3:10

Higgs couplings to gauge bosons + self-interactions.

$$\mathcal{L}_{\substack{higgs-\\gauge}} = \frac{1}{4} g^2 v^2 W_\mu^+ W^{-\mu} + \frac{1}{8}(g^2+g'^2)v^2 Z_\mu Z^\mu$$

$$\Rightarrow \frac{1}{4} g^2 v^2 \left(\frac{2vh}{v^2}\right) W_\mu^+ W^{-\mu} + \frac{1}{8}(g^2+g'^2)v^2 \frac{(2vh)}{v^2} Z_\mu Z^\mu$$

$$+ \frac{1}{4} g^2 v^2 \left(\frac{h^2}{v^2}\right) W_\mu^+ W^{-\mu} + \frac{1}{8}(g^2+g'^2)v^2 \left(\frac{h^2}{v^2}\right) Z_\mu Z^\mu$$

$$= 2\frac{m_W^2}{v} h W_\mu^+ W^{-\mu} + \frac{m_Z^2}{v} h Z_\mu Z^\mu$$

$$+ \frac{m_W^2}{v^2} h^2 W_\mu^+ W^{-\mu} + \frac{1}{2}\frac{m_Z^2}{v^2} h^2 Z_\mu Z^\mu$$

Higgs potential give Higgs mass + self-interactions.

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$$V(H) = -\mu^2 |H|^2 + \lambda |H|^4$$

Isolate cubic + quartic in second term, $H = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix}$,

$$v^2 = -\frac{\mu^2}{\lambda}.$$

$$\mathcal{L} \supset -V(H) = -\frac{\lambda}{4}(h+v)^4 + \ldots = -\frac{\lambda h^4}{4} - \lambda v h^3$$

Can use $m_h^2 = 2\lambda v^2$, $\quad -\lambda v h^3 = -\frac{m_h^2}{2v} h^3$.

Fermion gauge interactions: Follow from covariant derivative, familiar from QCD + QED.

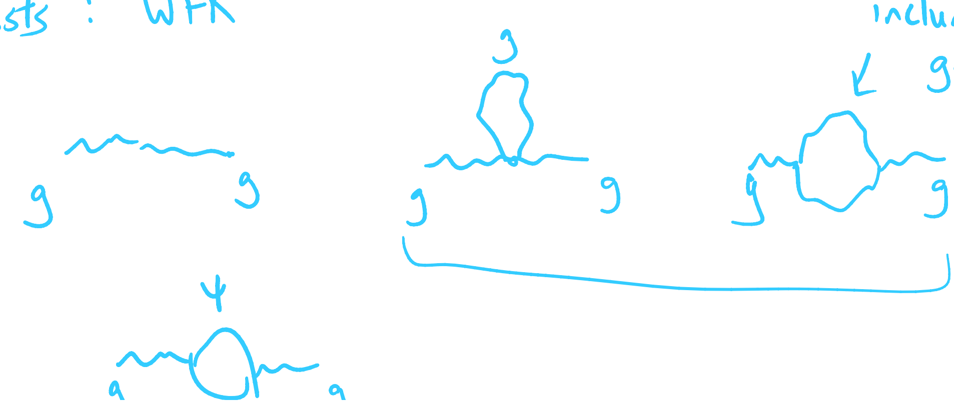
$$D_\mu = \partial_\mu - i g_s G_\mu^a t^a - \frac{ig}{\sqrt{2}}\left(W_\mu^+ T^+ + W_\mu^- T^-\right) - \frac{ig}{c_w} Z_\mu \left(T^3 - s_w^2 Q\right) - ie A_\mu Q$$

Lastly, self-interactions of gauge bosons (EW + gluons) also familiar Yang-Mills theory.

HW 1-1 is to do EW gauge self-interactions.

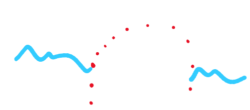
Done with tree level!

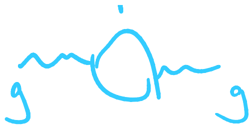
Ghosts: WFR

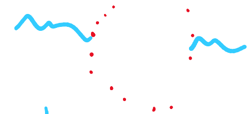


includes non-physical gauge boson polarization states in virtual loop

fix by ghosts







Cancel unphysical pols. of gluons.

Choice of R_ξ gauge,

$R_\xi \to \infty$, unitarity theory
$R_\xi \to 1$, Feynman 't-Hooft
$R_\xi \to 0$, Landau gauge

Account for massive physical dofs. of gauge bosons.

Ghosts remove unphysical gauge-dependent pol. in virtual loops.
Using R_ξ gauge-fixing, can mostly ignore ghost vertices.
(May need ghosts for Peskin-Takeuchi EW oblique parameters.]

Cheng & Li - ghost Lagrangian for EW theory.

$$\left(H \sim \begin{pmatrix} G^+ \\ \left(\frac{v+h}{\sqrt{2}}\right) + i G_0 \end{pmatrix} \right.$$

$$|D_\mu H|^2 \supset (\partial_\mu G^+) W^{-\mu}$$

+ gauge fixing to remove mixing.

μ  ν

$$\frac{-i}{p^2}\left(g^{\mu\nu} - \frac{(1-\xi)\, p^\mu p^\nu}{p^2 - \xi m_V^2} \right)$$

$$\xi \to \infty, \quad \frac{-i}{p^2}\left(g^{\mu\nu} - \frac{p^\mu p^\nu}{m_V^2} \right)$$

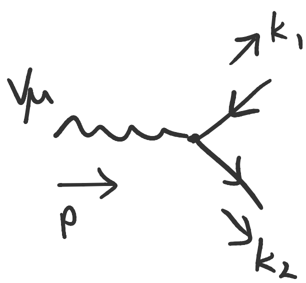
$$G^0, G^+ \quad \frac{i}{p^2 - \xi m^2} \to 0 \text{ for } \xi \to \infty$$

(Peskin 21.1)

Much of SM pheno is already accessible + calculable at tree-level.

Exercise: W + Z branching fractions.

In general, tree-level vector to 2 fermion decay.



$$i\left(g_V\gamma^\mu + g_A\gamma^\mu\gamma^5\right)$$

$$i\mathcal{M} = \bar{u}_{k_2} \cdot i\left(g_V\gamma^\mu + g_A\gamma^\mu\gamma^5\right) v(k_1)\,\epsilon_\mu(p)$$

$$|\mathcal{M}|^2 = \left(\bar{u}_{k_2}\, i\left(g_V\gamma^\mu + g_A\gamma^\mu\gamma^5\right) v_{k_1}\right)\epsilon_\mu(p) \cdot \left(\bar{v}_{k_1}\,(-i)\left(g_V\gamma^\nu + g_A\gamma^\nu\gamma^5\right) u_{k_2}\right)\epsilon^*_\nu(p)$$

$\bar\Psi\, i\not{\partial}\,\Psi$ is Hermitian

$Q\bar\Psi\, c\, \not{A}\,\Psi$ is Hermitian

$\uparrow$ must be real

$$|\mathcal{M}|^2 = \mathrm{Tr}\left[\left(g_V\gamma^\mu + g_A\gamma^\mu\gamma^5\right) v_{k_1}\bar{v}_{k_1} \left(g_V\gamma^\nu + g_A\gamma^\nu\gamma^5\right) u_{k_2}\bar{u}_{k_2}\right]\epsilon_\mu(p)\epsilon^*_\nu(p)$$

$$\sum_s v_{k_1}\bar{v}_{k_1} = \not{k}_1 - m\,\mathbb{1}$$

$$\sum_s u_{k_1} \bar{u}_{k_2} = \not{k}_2 + m_y \mathbb{1}$$

$$|\mathcal{M}|^2 = \mathrm{Tr}\Big[(g_V \gamma^\mu + g_A \gamma^\mu \gamma^5)(\not{k}_1 - m_x)$$
$$(g_V \gamma^\Pi + g_A \gamma^\Pi \gamma^5)(\not{k}_2 + m_y)\Big] \epsilon_\mu(p)\, \epsilon^*_\Pi(p)$$

$$= \mathrm{Tr}\Big[g_V \gamma^\mu \not{k}_1 g_V \gamma^\Pi \not{k}_2$$
$$+ g_A \gamma^\mu \gamma^5 \not{k}_1 g_V \gamma^\Pi \not{k}_2$$
$$+ \ldots \Big] \epsilon_\mu(p) \epsilon^*_\Pi(p)$$

$$= \Big\{ g_V^2 \mathrm{Tr}[\gamma^\mu \not{k}_1 \gamma^\Pi \not{k}_2]$$
$$+ \ldots \Big) \epsilon_\mu \epsilon^*_\Pi$$

$$= g_V^2 4 (k_1^\mu k_2^\Pi - k_1 \cdot k_2 g^{\mu\Pi} + k_1^\Pi k_2^\mu)$$
$$+ \ldots$$

Use * $\mathrm{Tr}[\gamma^\mu \gamma^\nu] = 4 g^{\mu\nu}$

$$\mathrm{Tr}[\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma]$$
$$= \mathrm{Tr}[-\gamma^\mu \gamma^\nu \gamma^\sigma \gamma^\rho +$$
$$\gamma^\mu \gamma^\nu \{\gamma^\sigma, \gamma^\rho\}]$$

$$\{\gamma^\alpha, \gamma^\beta\} = 2 g^{\alpha\beta}$$