QFT3

The Standard Model and EW Theory.

Lecture 2.

November 5, 2020.

Felix Yu

Last time: Motivation for QFT3

EW symmetry breaking + gauge boson masses.

Today: The SM at tree level.

Last time we reviewed the EW mixing angle that defines the change of basis from "gauge" basis of W^3_μ + B_μ to the mass basis of Z_μ + A_μ.

We have a similar story with EWSB and SM charged fermions.

From the Yukawa Lagrangian,

$$\mathcal{L}_{Yuk} = -y_u \bar{Q}_L \tilde{H} u_R - y_d \bar{Q}_L H d_R - y_e \bar{L}_L H e_R + h.c.$$

Again, $H = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix}$. $\bar{Q}_L = (\bar{u}_L \ \bar{d}_L)$, $\bar{L}_L = (\bar{\nu}_L \ \bar{e}_L)$.

and $\tilde{H} = \frac{1}{\sqrt{2}}\begin{pmatrix} v+h \\ 0 \end{pmatrix}$

Focusing only on vev, we get

$$\mathcal{L}_{Yuk}\Big|_v = -y_u \frac{v}{\sqrt{2}} \bar{u}_L u_R - y_d \frac{v}{\sqrt{2}} \bar{d}_L d_R - y_e \frac{v}{\sqrt{2}} \bar{e}_L e_R + h.c.$$

Recall that the Yukawa matrices are each general 3×3 complex matrices. Let us proceed and add the h.c. explicitly. Note that $\bar{\Psi}_L \Psi_R + h.c. = \bar{\Psi}_L \Psi_R + (\bar{\Psi}_L \Psi_R)^\dagger = \bar{\Psi}_L \Psi_R + \bar{\Psi}_R \Psi_L$

since $(\bar{\Psi}_L \Psi_R)^\dagger = \psi_R^\dagger (\psi^\dagger \gamma^0)^\dagger = \psi_R^\dagger \gamma^0 \psi = (P_R \psi)^\dagger \gamma^0 \psi = \psi^\dagger P_R \gamma^0 \psi$

$= \psi^\dagger \gamma^0 P_L \psi = \bar{\psi} \psi_L = \bar{\Psi}_R \Psi_L$

$$\mathcal{L}_{yuk}\Big|_v = -y_u^{ij}\frac{v}{\sqrt{2}}\bar{u}_L^i u_R^j - y_d^{ij}\frac{v}{\sqrt{2}}\bar{d}_L^i d_R^j - y_e^{ij}\frac{v}{\sqrt{2}}\bar{e}_L^i e_R^j$$
$$-(y_u^{ji})^*\frac{v}{\sqrt{2}}\bar{u}_R^j u_L^i - (y_d^{ji})^*\frac{v}{\sqrt{2}}\bar{d}_R^j d_L^i - (y_e^{ji})^*\frac{v}{\sqrt{2}}\bar{e}_R^j e_L^i$$

This is as far as we can go at the moment. To make further progress, we have to perform a change of basis from gauge basis to mass basis.

The starting point is to note that the SM without any Yukawa terms enjoys a $U(3)^5$ flavor symmetry, since each set of chiral fermions can be rotated into each other by a $U(3)$ transformation, leaving the overall Lagrangian unchanged.

e.g.
$$\bar{Q}_L i\not{\partial} Q_L = \bar{Q}_L U_Q^\dagger U_Q \, i\not{\partial} \, U_Q^\dagger U_Q Q_L$$
$$= \bar{\tilde{Q}}_L \, i\not{\partial}\, \tilde{Q}_L \text{, where } U_Q \in SU(3) \text{ matrix}$$
$$\text{and } \tilde{Q}_L \equiv U_Q Q_L .$$

There are 5 sets of fermions $(Q_L, u_R, d_R, L_L, e_R)$, so we have an overall $U(3)^5$ flavor symmetry. We decompose into $U(3) = U(1) \times SU(3)$ for each (the $U(1)$ factors are complicated to count – we will leave them for next time).

For the quarks these unitary rotations can be used to diagonalize the y_u and y_d matrices. The Leptons will also enjoy a diagonal y_e.

$$\mathcal{L}_{yuk}\Big|_v =$$
$$-\frac{v}{\sqrt{2}}\bar{u}_L V_{u_L}^\dagger V_{u_L} y_u V_{u_R}^\dagger V_{u_R} u_R - \frac{v}{\sqrt{2}}\bar{d}_L V_{d_L}^\dagger V_{d_L} y_d V_{d_R}^\dagger V_{d_R} d_R$$
$$-\frac{v}{\sqrt{2}}\bar{e}_L V_{e_L}^\dagger V_{e_L} y_e V_{e_R}^\dagger V_{e_R} e_R + \text{h.c.}$$
$$\equiv -\frac{v}{\sqrt{2}}\bar{u}_L^m y_u^{diag} u_R^m - \frac{v}{\sqrt{2}}\bar{d}_L^m y_d^{diag} d_R^m$$
$$-\frac{v}{\sqrt{2}}\bar{e}_L^m y_e^{diag} e_R^m + \text{h.c.}$$

The diagonal structure guarantees the vanishing of off-diagonal entries, so we identify $\frac{v}{\sqrt{2}}\,\mathrm{diag}(y_u^{11}, y_u^{22}, y_u^{33})$ as the up, charm, and top masses (order increasing WLOG).

Same for down-type quarks + leptons.

We get

$$= -\frac{v}{\sqrt{2}} y_u^{diag} (\bar{u}_L^m u_R^m + \bar{u}_R^m u_L^m) - \frac{v}{\sqrt{2}} y_d^{diag} (\bar{d}_L^m d_R^m + \bar{d}_R^m d_L^m)$$

$$- \frac{v}{\sqrt{2}} y_e^{diag} (\bar{e}_L^m e_R^m + \bar{e}_R^m e_L^m)$$

Note $\bar{u}_L^m u_R^m + \bar{u}_R^m u_L^m = \bar{u}(P_L + P_R)u = \bar{u}u$.

$$= -\frac{v}{\sqrt{2}} y_u^{diag} \bar{u}u - \frac{v}{\sqrt{2}} y_d^{diag} \bar{d}d - \frac{v}{\sqrt{2}} y_e^{diag} \bar{e}e.$$

We have thus used the global symmetry to choose $V_{u_L}, V_{u_R}, V_{d_L}, V_{d_R}, V_{e_L}, V_{e_R}$ s.t. $y_u^{diag} = V_{u_L} y_u V_{u_R}^\dagger$, etc.

Remark: The original y_u, y_d, y_e are always arbitrary!

But they define specific V matrices.

Then, we do the change of basis on the spinors to get to the mass basis: $V_{u_R} u_R \equiv u_R^m$, $V_{u_L} u_L \equiv u_L^m$, etc.

Note that we needed 6 Vs to get all Yukawa matrices diagonal. So the $\bar{Q}_L i\cancel{D} Q_L$ gauge interactions will now be <u>non-diagonal</u>.

Explicitly,

$$\bar{Q}_L i\cancel{D} Q_L = \bar{u}_L i\cancel{\partial} u_L + \bar{u}_L g_s \cancel{G}^a t^a u_L + \bar{u}_L \frac{g}{c_W} \cancel{Z} (T^3 - s_W^2 Q) u_L + \bar{u}_L e A_\mu Q u_L$$

$$+ (\text{same for } d_L)$$

$$+ \bar{u}_L \frac{g}{\sqrt{2}} \cancel{W}^+ d_L + \bar{d}_L \frac{g}{\sqrt{2}} \cancel{W}^- u_L$$

Adopting mass basis: first (two) rows

$$= \bar{u}_L V_{u_L}^\dagger V_{u_L} i\cancel{\partial} V_{u_L}^\dagger V_{u_L} u_L \to \bar{u}_L^m i\cancel{\partial} u_L^m$$

So all neutral current interactions remain diagonal and flavor conserving & in the mass basis.
On the other hand, the $W^{+/-}$ interactions are now flavor violating.

$$\bar{u}_L V_{u_L}^\dagger V_{u_L} \frac{g}{\sqrt{2}} \not{W}^+ V_{d_L}^\dagger V_{d_L} d_L + h.c.$$
$$= \bar{u}_L^m V_{u_L} V_{d_L}^\dagger \frac{g}{\sqrt{2}} \not{W}^+ d_L^m + h.c.$$

This matrix $V_{u_L} V_{d_L}^\dagger \equiv V_{CKM}$ is the only source of flavor violation in quark interactions.
(For nonvanishing neutrino masses, also get PMNS matrix $\exists$ will discuss when we get to neutrino physics).

So Feynman rules for charged current exchange must include CKM factor.

-- - - -

Higgs Yukawa + gauge + self interactions.

Easiest way to derive Higgs couplings is a pseudo-spurion analysis, by replacing $v \to v+h$.

Then, Yukawa couplings are

$$\mathcal{L}_{Yuk}\Big|_h = -\frac{h}{\sqrt{2}} \bar{u}u\, y_u^{diag} - \frac{h}{\sqrt{2}} y_d^{diag} \bar{d}d - \frac{h}{\sqrt{2}} y_e^{diag} \bar{e}e$$

$$\mathcal{L}_{\substack{\text{higgs-}\\\text{gauge}}} = \frac{1}{4} g^2 v^2 W_\mu^+ W^{-\mu} + \frac{(g^2+g'^2)}{8} v^2 Z_\mu Z^\mu$$
$$\rightsquigarrow \frac{1}{4} g^2 (v+h)^2 W_\mu^+ W^{-\mu} + \frac{1}{8}(g^2+g'^2)(v+h)^2 Z_\mu Z^\mu$$
$$\supset \frac{1}{4} g^2 v^2 \cdot \frac{2h}{v} W_\mu^+ W^{-\mu} + \frac{1}{8}(g^2+g'^2) v^2 \frac{2h}{v} Z_\mu Z^\mu$$
$$+ \frac{1}{4} g^2 v^2 \frac{h^2}{v^2} W_\mu^+ W^{-\mu} + \frac{1}{8}(g^2+g'^2) v^2 \frac{h^2}{v^2} Z_\mu Z^\mu$$
$$= \frac{2h}{v} m_W^2 W_\mu^+ W^{-\mu} + \frac{h}{v} m_Z^2 Z_\mu Z^\mu + \frac{h^2}{v^2} m_W^2 W_\mu^+ W^{-\mu} + \frac{h^2}{2v^2} m_Z^2 Z_\mu Z^\mu$$

Higgs potential gave Higgs mass + cubic + quartic self-interactions

$$V(H) = \mu^2 |H|^2 + \lambda |H|^4$$

$$v^2 = -\frac{\mu^2}{\lambda} \quad , \quad H = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v+h \end{pmatrix}$$

Extract cubic coupling: $\frac{\lambda}{4} \cdot (v+h)^4 \Rightarrow 4\frac{\lambda}{4} h^3 v + \frac{\lambda}{4} h^4$
+quartic

$$= \lambda v h^3 + \frac{\lambda}{4} h^4$$

Since $m_h^2 = 2\lambda v^2$, can also write

$$\mathcal{L}_{scalar} \supset -\frac{m_h^2}{2v} h^3 - \frac{m_h^2}{2v^2} h^4$$

Fermion gauge interactions follow from covariant derivative, should be familiar from QCD + QED.

$$D_\mu = \partial_\mu - i g_s G_\mu^a T^a - \frac{ig}{\sqrt{2}} \left(W_\mu^+ T^+ + W_\mu^- T^- \right)$$
$$- \frac{ig}{c_W} Z_\mu \left(T^3 - s_W^2 Q \right) - ie A_\mu Q$$

EW gauge self-interactions + gluon self-interactions are also familiar from non-Abelian gauge theory.
EW gauge self-interactions are HW 1, problem 1.1.

With our tree-level knowledge of SM, can already do some non-trivial basic SM pheno.

Exercise: W + Z branching fractions.

Next time: Axial-vector couplings + unitarity. Gauge boson scattering + unitarity. Adding new gauge bosons. (W', Z')