Lecture 19, February 10, 2021

Wednesday, February 10, 2021 2:16 PM

Last time: DM EFT + EFT validity + closure test

DM simplified models + UV (in)completeness

Today: Neutrino physics

Besides HQET or SCET, the most prevalent use of effective operators in SM physics is in the context of neutrinos.

Neutrino oscillation: Super-K in 1995 $\Rightarrow$ ν's must have mass

Open question: what is the nature of ν mass?

i.e. Dirac vs. Majorana

Actual physical masses are parameters to measure.

Also, PMNS entries (equivalent to CKM but for leptons).

Address first question:

Compare + contrast quarks vs. leptons in SM:

$$Q = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \quad \mathcal{L}_q = -Y_u \bar{Q} \tilde{H} u_R - Y_d \bar{Q} H d_R + h.c.$$

$$\overset{\Rightarrow}{EWSB} \quad -m_u \bar{u} u - m_d \bar{d} d$$

(rotation matrices create $V_{CKM} = V_u V_d^\dagger$)

$$L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \quad \mathcal{L}_\ell = -Y_\nu \bar{L} \tilde{H} N_R - Y_e \bar{L} H e_R + h.c.$$

$$\overset{\Rightarrow}{EWSB} \quad \underline{-m_\nu \bar{\nu} \nu} \quad -m_e \bar{e} e$$

$+$ PMNS

(no PMNS, $U(1)_e \times U(1)_\mu \times U(1)_\tau$

EWSB

$+$ PMNS

$U(1)_e \times U(1)_\mu \times U(1)_\tau$ flavor conservation)

But the "natural" term analogous to up quark masses.

Suppose we adopt the most naive approach:

Include 3 N_R = SM gauge singlets.

Then, we have to diagonalize both Y_ν + Y_e, with only $U(3)^3$ symmetries for

$U(3)^3 : U(3)_L \times U(3)_{e_R} \times U(3)_{N_R}$

$\Rightarrow$ Consequently, W interactions now violate flavor & get PMNS matrix + $U(1)_L$ is remaining global symmetry.

$$\frac{g}{\sqrt{2}}(\bar{\nu}_1 \ \bar{\nu}_2 \ \bar{\nu}_3) W^+ V_{PMNS} \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}$$

$V_{PMNS} \neq \mathbb{1}$ Derivation exactly same PMNS.

$V \sim \begin{pmatrix} 0(.3) - 0(0.5) \\ \text{in all} \\ \text{entries} \end{pmatrix}$

PMNS is $\mathbb{1}$ if ν-masses are turned off.

Why not be happy with this?

1. What numerical values of Y_ν are needed?
2. Are all global symmetries exercised / exhausted?

Ans. to #1 : $m_\nu \sim$?

Cosmology : $\sum m_\nu \lesssim 0.3\ eV$ (CMB, Planck)
There is model-dependence.

From atmos. neutrino oscillation

$\Delta m^2_{32} \simeq 2.5 \cdot 10^{-3}\ eV^2$, $\theta_{23} \sim 45°$

From solar neutrino oscillation

$\Delta m^2_{21} \simeq 7.4 \cdot 10^{-5}\ eV^2$, $\theta_{12} \simeq 33°$

Reactor neutrino expts, $\theta_{13} \sim 9°$

So, if we set $m_\nu = 0.1\ eV$,

then $y_\nu \sim \frac{0.1\ eV}{246\ GeV} \sim \frac{0.1\ eV}{100\ GeV}\ 10^{-12}$

Compare to the smallest & largest known Yukawas,

$y_e \sim \frac{511\ keV}{246\ GeV} \sim 10^{-6}$, $y_t \sim \frac{174\ GeV}{246\ GeV} \sim 1$

Dirac neutrino masses extend flavor hierarchy by another 6 orders of magnitude.

Ans. to #2.

In principle, how do you write the SM Lagrangian?

[Given fields, gauge symmetries + charges, write all allowed terms at the renormalizable level.

Global syms. are then always accidental!

This distinguishes the (A) ν_R-less SM + Dirac N_R

vs. the (B) SM + N_R Lagrangians!

In the second case:

$$\mathcal{L}_{N_R} = \bar{N}_R i \not{\partial} N_R - m (\bar{N}_R)^c N_R$$ in addition to the Y_ν term from before.

$\bar{N}_R i \not{\partial} N_R$: new kinetic term

$m (\bar{N}_R)^c N_R$: Majorana mass term since N_R is SM gauge singlet.

Minimal interactions (A) $$\mathcal{L}_{tot} = \mathcal{L}_{SM} + \bar{N}_R i \not{\partial} N_R - (Y_\nu \bar{L} \tilde{H} N_R + h.c.)$$

More general given same rules as $\mathcal{L}_{SM}$ (B) $$\mathcal{L}_{tot} = \mathcal{L}_{SM} + \bar{N}_R i \not{\partial} N_R - m (\bar{N}_R)^c N_R - (Y_\nu \bar{L} \tilde{H} N_R + h.c.)$$

(B) is renormalizeable. The relevant phenomenology for ν physics tests scales $\ll m \sim O(10^{14})$ GeV

What is right approach between A + B?

Usually favor minimality but also prize quantum completeness = "everything not forbidden in quantum physics is allowed."

(A) is more minimal.

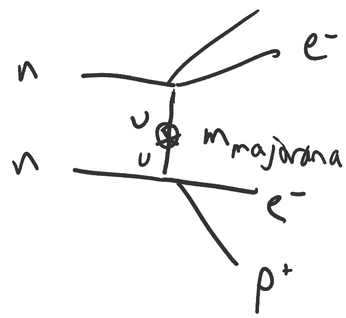
(B) is more complete.

To favor (A), need to "enforce" global symmetry for lepton #. This is first case we have where this question arises, since ν's are first particles that can be Majorana.

Global symmetry is lepton number:

Expt. to test lepton # violation by 2 units is $0\nu 2\beta$ decay: (neutrinoless double beta decay.)

p^+



No ν in final state, but $2\,e^-$.

$\# L_{tot} = 0$ initial

$\# L_{tot} = 2$ final.

Not observed (yet), but actively searched.

ν_R-less $\mathcal{L}_{SM}$ as known now predicts $L\#$ conservation.

Given that we need to have ν-masses, necessarily need ν_R (at some scale) $\Rightarrow$ choose (A) or (B).

(A) ↑ preserve $L\#$ cons.

(B) ↑ violate $L\#$ by 2 units

Break until 3:20pm.

Critical question: Are neutrinos their own anti-particle?

Charge conjugation operator $\hat{C}$: $\psi^c = C\bar{\psi}^T$

based on MITP SS 2017 by Akmehdov

where $C = i\gamma^2\gamma^0$ in Weyl represent.

$$C^{-1}\gamma_\mu C = -\gamma_\mu^T$$

$$C^+ = C^{-1} = -C^* \qquad (C^T = -C).$$

$$* \quad (\psi_L)^c = (\psi^c)_R \quad , \quad (\psi_R)^c = (\psi^c)_L$$

Default notation is LHS of these.

Normal Dirac mass: $\psi = \begin{pmatrix} \eta \\ \xi \end{pmatrix}$ ← 2-comp. Weyl spinors

Normal Dirac mass: $\Psi = \begin{pmatrix} \eta \\ \xi \end{pmatrix}$ spinors

$$\mathcal{L} = m\underbrace{\bar{\Psi}\Psi} = m\left(\bar{\eta}\xi + \bar{\xi}\eta\right)$$
$$= m\bar{\Psi}\,\mathbb{I}\,\Psi$$
$$= m\bar{\Psi}\left(P_L + P_R\right)\Psi$$
$$= m\left(\bar{\Psi}P_L\Psi + \bar{\Psi}P_R\Psi\right)$$
$$= m\left(\bar{\xi}\eta + \bar{\eta}\xi\right)$$

Contrast with Majorana mass:

Require $\xi = (\eta)^c$, then

$$\mathcal{L} = \underline{m\bar{\Psi}\Psi} = m\left(\underbrace{\overline{(\eta^c)}}\eta + \bar{\eta}(\eta)^c\right)$$

$\bar{\Psi} = \Psi^\dagger\gamma^0$

$$= m\left((\eta^c)^\dagger\gamma^0\eta + \bar{\eta}(\eta)^c\right)$$

$\Psi^c = C(\bar{\Psi})^T$

$$= m\left(\left(C(\bar{\eta})^T\right)^\dagger\gamma^0\eta + \bar{\eta}(\eta)^c\right)$$

$\underbrace{\overline{(\eta^c)} = -\eta^T C^{-1}}$

$$= m\left(\left(C(\eta^\dagger\gamma^0)^T\right)^\dagger\gamma^0\eta + \bar{\eta}(\eta)^c\right)$$

$C = i\gamma^2\gamma^0$

$$= m\left(\left(C\gamma^0\eta^*\right)^\dagger\gamma^0\eta + \bar{\eta}(\eta)^c\right)$$

$C^{-1} = C^\dagger = -C^*$

$$= m\left(\eta^T\gamma^{0\dagger}C^\dagger\gamma^0\eta + \bar{\eta}(\eta)^c\right)$$
$$= m\left(\eta^T\gamma^{0\dagger}(i\gamma^2\gamma^0)^\dagger\gamma^0\eta + \bar{\eta}(\eta)^c\right)$$

$i(\gamma^2)^\dagger\gamma^0$

$= -i\gamma^0(\gamma^2)^\dagger$

$$= m\left(-\eta^T\cancel{\gamma^{0\dagger}\gamma^0}\underbrace{i(\gamma^2)^\dagger}\gamma^0\eta + \bar{\eta}(\eta)^c\right)$$

$= m\left(\ \ \eta^T C^{-1} \ \ldots\right)$

$$m\,(-\eta \quad \cancel{0} \; 0 \; \underbrace{(0 \;\; 0)}\; \eta \; + \eta \; (\eta))$$

$$= -i\,\gamma^0 (\gamma^2)^\dagger$$

$$C^\dagger = (i\gamma^2\gamma^0)^\dagger$$

$$= -i\,(\gamma^0)^\dagger (\gamma^2)^\dagger$$

$$= m\left(-\eta^T C^{-1}\eta + \underbrace{\bar{\eta}\,(\eta)^c}_{\text{h.c. of first term}}\right)$$

$$m\bar{\Psi}\Psi = 2m\;\bar{\eta}(\eta)^c$$

$\Rightarrow$ Mass term for Weyl fermion by itself <u>is</u> possible with requirement that $\Psi = \begin{pmatrix} \eta \\ (\eta)^c \end{pmatrix}$

$2m$ is simply dof. counting.

Dirac = 2 Weyl fermions

Majorana = (LH) projection of Dirac fermion = 1 Weyl fermion

Aside: What other SM particles can be own antiparticle?

Mostly not, because of charge.

SU(2) is broken $\Rightarrow$ Higgs boson is own antiparticle.

Try to understand Majorana vs. Dirac via scalar field theory, Higgs is <u>bad</u> example because EW symmetry is still linear/non-linearly realized.

Analogy between Dirac + Majorana fermions + gauge singlet real scalars + Higgs scalars

(Instructive because dofs. + symmetries match precisely.)

$$\mathcal{L}_{Maj} = -\frac{m}{2}\,\overline{(N_R)^c}\,N_R$$

$$\mathcal{L}_{Maj} = -\frac{m}{2}\,\overline{(N_R)^c}\,N_R$$

No control over scale of m. Free param.

Extract light neutrino masses.

Mass matrix $\sim \begin{pmatrix} 0 & y_\nu v \\ y_\nu v & m \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R \end{pmatrix}$

Seesaw mass matrix.

Get light neutrino masses $m_{light} \sim \frac{(y_\nu v)^2}{m}$

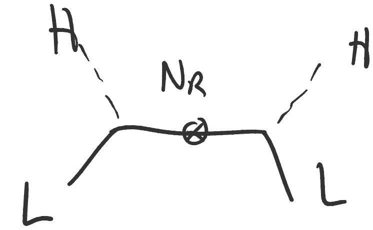
Gives correct neutrino masses for $m \sim 10^{14}$ GeV, $y_\nu \sim O(1)$

but lots of model freedom about scales + params.

Alternatively, treat ν-mass problem via effective operator. No N_R in theory: go to dim-5 Weinberg operator. Schematically: $\mathcal{O}_\nu = \frac{LL\,HH}{\Lambda}$

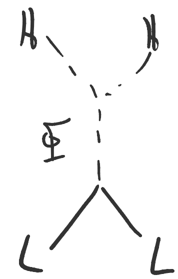
Distinct UV completions:

N_R prop scales as $\frac{1}{M_R}$



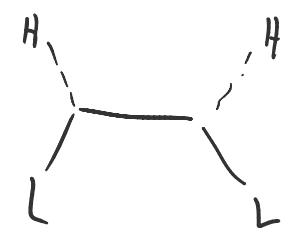
Type-1 seesaw

$\frac{(LH)^c(LH)}{\Lambda}$



Type-II

SU(2) triplet scalar



Type III

Fermion triplet

Λ

slalar

$$\underbrace{(\bar{L}^c \sigma^a L)(H\sigma^a H)}_{\Lambda}$$

$$\mathcal{L} = |D_\mu H|^2$$

$$\ni \left(g\sigma^a W^a_\mu H\right)^\dagger \left(g\sigma^b W^b_\mu H\right)$$

$$\ni \left(\underbrace{\cancel{g\sigma^a} W^a_\mu v}_{2}\right) \left(\underbrace{\sigma^b W^b_\mu}_{\underbrace{3}_{\Phi}} \underbrace{H}_{2}\right) \Rightarrow \mathcal{L} = \lambda\left(H_a \Phi_{ab} H_b + h.c.\right)$$