Lecture 19.
QFT III: The SM and EW Theory.
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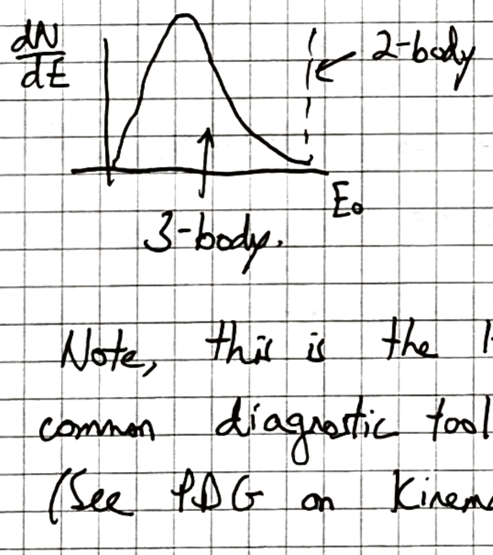
Last time: EFT validity & closure test
DM simplified models & UV (in) completeness
This time: Neutrino physics (based on Akhmedov lectures on Neutrino physics @ MITP, 2017)

The most widespread use of EFT operators to study particle physics today is in neutrino physics (or maybe HQET or SCET.)

History: From observations of beta decay,
$A(Z,N) \to A(Z\pm 1, N\mp 1) + e^{\mp}$; the two-body decay predicts an exact energy for the outgoing electron. Instead, a distribution was observed:



The third body was hypothesized as the neutrino by Pauli in 1930.

Note, this is the 1-D projection of the more common diagnostic tool known as the Dalitz Plot. (See PDG on Kinematics.)

Ever since the discovery of neutrino oscillations in 1995 by Super-K, the field of neutrino physics has progressed rapidly to measure all possible neutrino properties.

The state of the art:

We know the PMNS mixing angles $(\theta_{12}, \theta_{23}, \theta_{13})$ but only weak bounds on the Dirac phase δ and no information on the possible Majorana phases. We also know the two squared mass differences but not the neutrino hierarchy nor the individual neutrino masses.

PDG Planck CMB data : $\sum m_\nu < 1.1 eV$ @ 90%

$$\Delta m^2_{21} = (7.53 \pm 0.18) \cdot 10^{-5}\, eV^2$$
$$|\Delta m^2_{32}|^* = (2.453 \pm 0.034) \cdot 10^{-3}\, eV^2$$
$$\sin^2\theta_{12} = 0.307 \pm 0.013$$
$$\sin^2\theta_{23} = 0.545 \pm 0.021$$
$$\sin^2\theta_{13} = 0.0218 \pm 0.0007$$

How does this data get accomodated in the SM?

There are two approaches.

(A) ~~The~~ Take the SM as given and add the minimal interaction that gives ν-masses.

(B) Revisit the SM field content & write a new RH neutrino field and a new renormalizable SM. Lagrangian.

We will see that the difference between the two approaches are precisely the question of whether neutrinos are Majorana.

A. In this case, we follow the analogy with the quarks: in order to give masses to each component of an $SU(2)$ doublet, we need to have RH fields for both upper & lower $SU(2)$ fields $\Rightarrow$

$$\mathcal{L} = -y_u \bar{Q}\tilde{H} u_R - y_d \bar{Q} H d_R$$

$$\underbrace{-y_\nu \bar{L}_L \tilde{H} N_R}_{\text{new term}} - y_e \bar{L}_L H e_R + h.c.$$

new term. new field

This is the minimal interaction that is needed to give masses to neutrinos. After EWSB, we then diagonalize y_ν and y_e using $SU(3)_{e_R} \times SU(3)_{N_R} \times SU(3)_{e_L} \times SU(3)_{\nu_L}$ global symmetries. The $U(1)$ symmetries are also broken to a global $U(1)_L$ (lepton number) conservation number, instead of separate $U(1)_e \times U(1)_\mu \times U(1)_\tau$ global numbers. In exact analogy to quarks, the "extra" $SU(3)_{\nu_L} \times SU(3)_{e_L}$ symmetry we need creates a distinction in the W-charged current interactions and the mass basis: This is known as the Pontecorvo-Maki-Nagawa Sakata (PMNS) matrix, $V_{PMNS} = U_{\nu_L} U_{e_L}^\dagger$.

$$\mathcal{L} \supset \frac{g}{\sqrt{2}} \bar{\nu}_L W^+ V_{PMNS}\, e_L + h.c.$$

The resulting Dirac neutrino masses provide a minimal implementation of neutrino masses. The only price is the necessity of extending the fermion mass hierarchy by another six orders of magnitude.

$m_t \sim 173\,\text{GeV} \Rightarrow y_t \sim \mathcal{O}(1)$

$m_e \sim 511\,\text{keV} \Rightarrow y_e \sim 10^{-6}$

$m_\nu \sim 0.1\,\text{eV} \Rightarrow y_\nu \sim 10^{-12}$

Aesthetically, the fermion mass hierarchy is a bias that all parameters should be $\mathcal{O}(1)$, unless protected by a symmetry. Given that the Yukawas signal the breaking of chiral symmetry, they are technically ~~symmetry~~ natural.

(B) In this approach, we add a RH neutrino field from the beginning. As a SM gauge singlet, the new Lagrangian terms are

$$\mathcal{L} \supset \bar{N}_R i\not{\partial} N_R - m_N \bar{N}_R^c N_R - \left(y_\nu \bar{L}_L \tilde{H} N_R + h.c. \right)$$

The mass term $m_\nu \bar{N}_R^c N_R$ is the Majorana mass term, the first time such a term can be written down. Majorana masses are only allowed when the field is neutral, since they must be their own charge conjugate.

In terms of global symmetries, the Majorana mass term leads to a breaking of $U(1)_L$ by 2 units.

We thus have the possibility of both Majorana + Dirac mass terms:

$$m_\nu = \begin{pmatrix} 0 & y_\nu v \\ y_\nu v & m_N \end{pmatrix}$$

with each as 3×3 flavor space blocks.

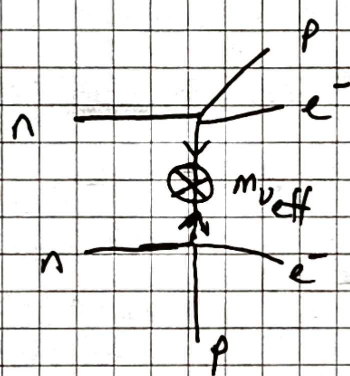
To get the right masses, for $y_\nu v \sim 100$ GeV, the light neutrino masses are

$$m_\nu \sim \frac{(100\text{ GeV})^2}{m_N} \sim 0.1\,\text{eV}$$

$$\Rightarrow \frac{(100\text{ GeV})^2}{0.1\text{ eV}} \sim m_N \sim 10^4 \cdot \text{GeV} \cdot 10^{10} \sim 10^{14}\text{ GeV}$$

This is called the seesaw mechanism, with $m_N \sim 10^{14}$ GeV & $m_{Dirac} \sim 100$ GeV giving rise to $\mathcal{O}(0.1)$ eV light neutrinos.

The effective Majorana mass for the neutrinos gives rise to a unique signal for testing the Majorana nature of neutrinos, which is neutrino-less double-β decay:

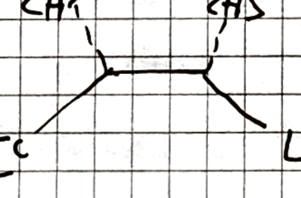


If no neutrinos are observed, we have a $\Delta L = 2$ process from $2n \to 2p + 2e^-$.

Critically, neutrinos are the only candidates where they can be their own antiparticle.

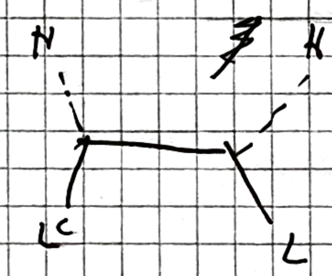
To treat model (B) as an effective operator, we can integrate out the RH neutrinos to generate the operator

$$\mathcal{L} = \frac{(LH)^c(LH)}{\Lambda}, \text{ with } \Lambda \sim m_N \sim 10^{14}\text{ GeV}.$$



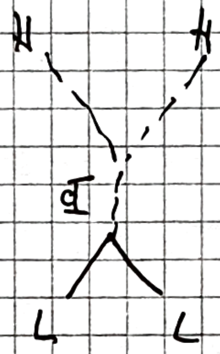
After EWSB, the light neutrinos have $\mathcal{O}(0.1)$ eV masses.

Finally, we note that the $\frac{(LH)(LH)}{\Lambda}$ seesaw operator has 3 possible tree-level UV completions:



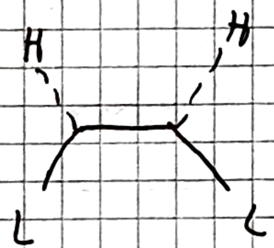
$\frac{(\bar{L}^c H)^c LH}{\Lambda}$

Type-I seesaw
with N_R
EW fermion
singlet



$\frac{(\bar{L}^c \sigma^a L)(H\sigma^a H)}{\Lambda}$

Type-II
SU(2) triplet
scalar



$\frac{(\bar{L}\sigma^a H)^c (L\sigma^a H)}{\Lambda}$

Type-III
SU(2) triplet
fermion