Lecture 15, January 21, 2021

Thursday, January 21, 2021 2:19 PM

Last time: 1-loop SM, weak sector
FCNCs at 1-loop.

Qualitatively new phenomena @ 1-loop vs. tree-level.

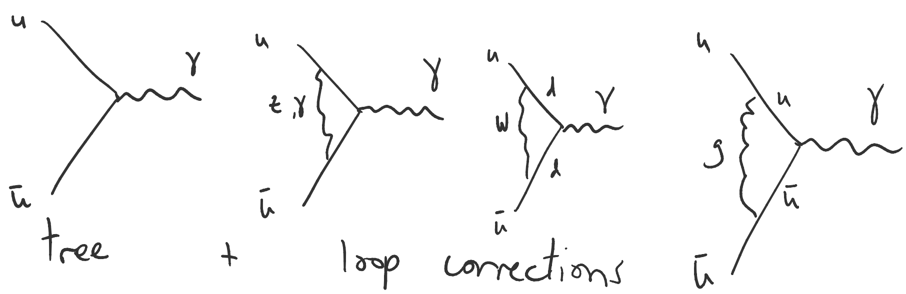
1-loop correction:

Ⓐ Either going loop-level correction existing tree-level vertex → important for precision, qualitatively same.

Ⓑ Have new process w/ no tree-level counterpart.

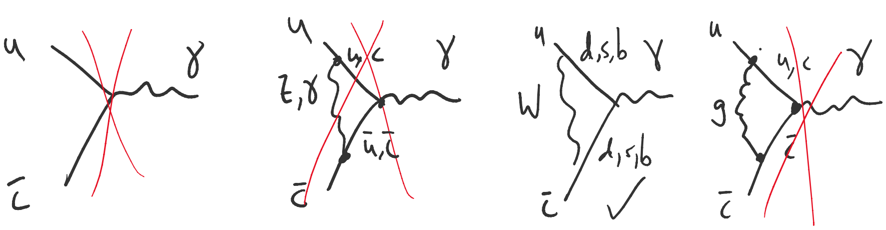
Examples in Higgs physics: $h \to gg$, $h \to \gamma\gamma$, $h \to Z\gamma$

Critical since ggF dominates over tree-level.



tree + loop corrections

Distinction was when tree-level does not exist

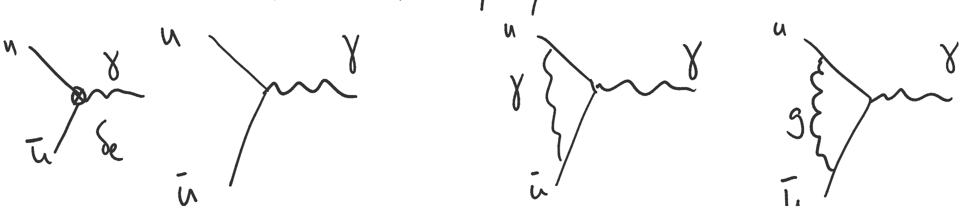


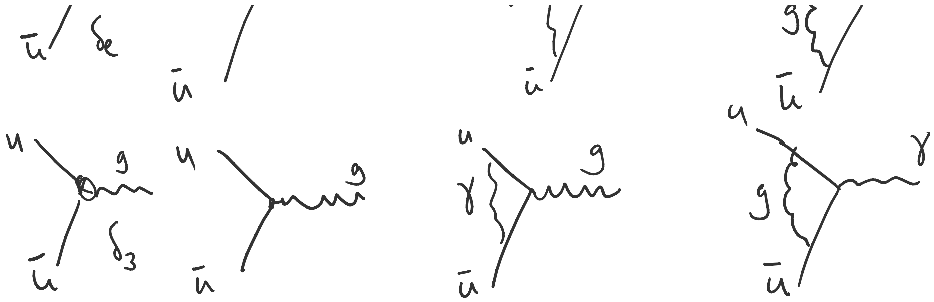
Main issue to consider: UV divergence.

Ⓑ UV divergence in FCNC loop controlled by unitary matrix CKM.

Ⓐ Divergence in Flavor-conserving. Use counterterms same as QED renormalization. CTs get mixed-order contributions.

To see this, simplify to QCD + QED:



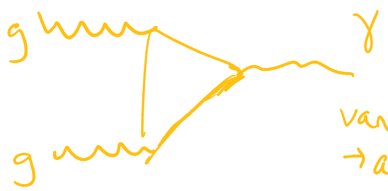


With just QCD or QED ($e \to 0$ or $g_s \to 0$), easy since 3 loops vanish. Both couplings non-zero; δ_e + δ_3 defined to capture sums of UV divergences. Phenomenon of mixed gauge corrections.

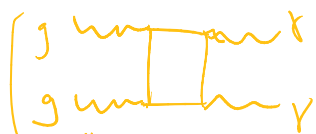
Consequence: gauge couplings have mixed RGE running.

$$\beta(e) \sim e + e^3 + \dots + e g_s^2 + \dots$$

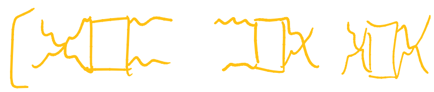
$$\beta(g_s) \sim g_s + g_s^3 + \dots + g_s e^2 + \dots$$



vanishes.
→ anomaly cancellation
↳ consistency condition on the independence of an amplitude on momentum shifts of loop momenta



$\int d^4k \frac{1}{(k)^4} \sim \ln \Lambda_{UV}$



↳ cancels divergence

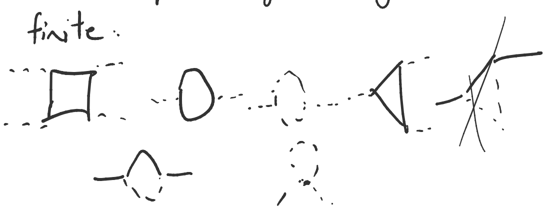
$\int d^4k \frac{1}{(k)^4} \cdot \frac{1}{k^2}$

$\int d^4k \frac{1}{(k)^5}$

This diagram corresponds to $\frac{1}{\Lambda^4} F_{\mu\nu} F_{\rho\sigma} G^a_{\mu\nu} G^a_{\rho\sigma}$? i.e. not a dim.-4 $\mathcal{L}$ term.

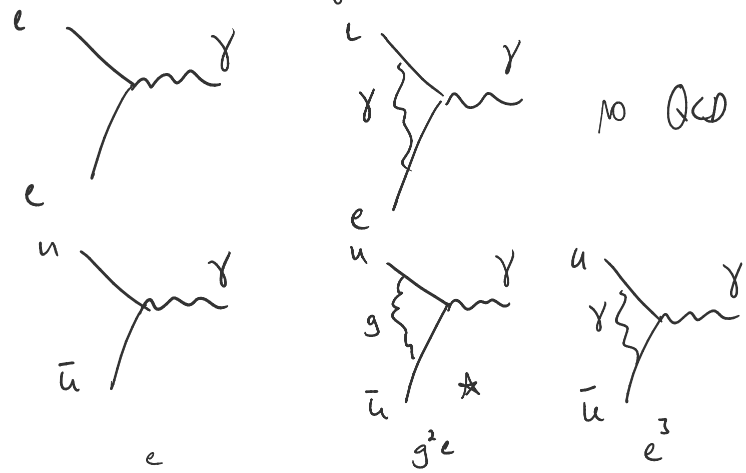
$\Rightarrow G^a_\mu G^a_\nu A_\mu A_\nu$ is not gauge invariant.

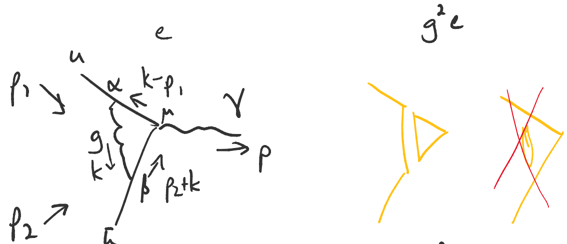
Set of 1-loop divergent diagrams is finite.



Anomaly cancellation: For chiral representations, loop integral is generally not invariant under shifts of loop momenta. Instead, set of fermions need to have a prescribed set of charges + reps. such that the shift, after summing over all fermions, gives no net change.

after switching one of fermions, gives lower charge.



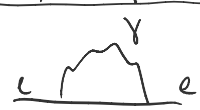


$$iM = \int \frac{d^4k}{(2\pi)^4} \bar{v}_2 \cdot i g_s t^a \gamma^\beta \cdot \frac{i(\not{p}_2+\not{k})}{(p_2+k)^2} \cdot ie\gamma^\mu$$

$$\cdot \frac{i(\not{k}-\not{p}_1)}{(k-p_1)^2} \cdot i g_s t^a \gamma^\alpha \cdot u_1 \cdot \frac{-i g_{\alpha\beta}}{k^2}$$

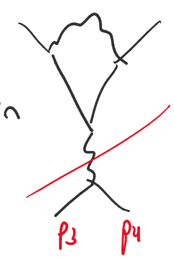
$$= \int \frac{d^4k}{(2\pi)^4} \bar{v}_2 \frac{\gamma^\beta (\not{p}_2+\not{k}) \gamma^\mu (\not{k}-\not{p}_1) \gamma^\alpha u_1}{k^2 (k+p_2)^2 (k-p_1)^2} \cdot g_s^2 e$$

Mixed gauge 1-loop corrections



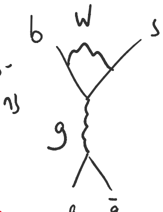
Physical result of FCNC is "off-shell" or Q^2-dependent external vector. Extension to 4-fermion like penguin diagram are motivated.

EW penguin



FCNC vtx
⊕ tree γ/Z vtx
$(p_3+p_4)^2 = Q^2$ probing FCNC.

Chromo-penguins



Other category of 1-loop structure of SM is
the general form-factor behavior of gauge bosons.

Also do Flavor-conserving to flavor-conserving.

Essentially, 1-loop treatment of fermi interaction.

Recall: $J_{W^+_\mu} \otimes J_{W^-_\mu}$ = Fermi interaction
became set of dim.-6 operators after integrating out W-boson.

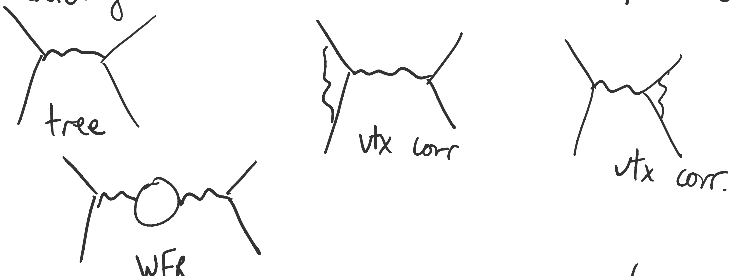
At 1-loop: loop on W-vtx ⊗ W-tree ⇒ loop correction to existing process.

Put loop on vector boson itself. Analyzing wavefcns. of vector bosons @ finite Q^2.

Called EW Oblique corrections.

@ 1-loop ⇒ Renormalization conditions fix SM at given Q^2 (usually at pole mass of external states), but form factor was predicted precisely at given loop order.

4-Fermi scattering

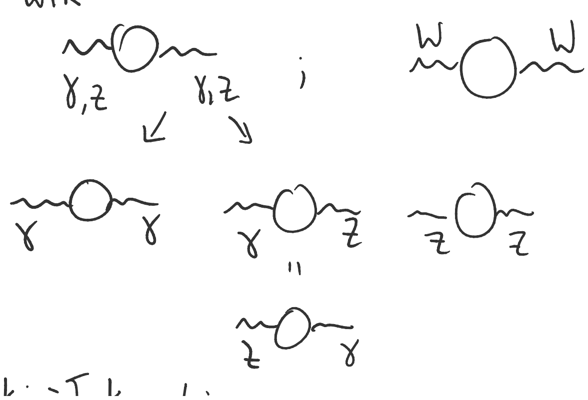


exp.: initial state are e^+e^-: Can control $(p_1+p_2)^2 = s$.
Access WFR for given $s = Q^2$.

Choose special set of Q^2. Supposed to probe finite effects from heavy new physics.

Peskin-Takeuchi.

Set of WFR



Schwartz, Section. 31.1.2.

Peskin-Takeuchi:

$$S \equiv \frac{4c^2s^2}{\alpha_e}\left[\frac{\Pi^{new}_{ZZ}(m_Z^2)-\Pi^{new}_{ZZ}(0)}{m_Z^2} - \frac{c^2-s^2}{cs}\frac{\Pi^{new}_{Z\gamma}(m_Z^2)}{m_Z^2} - \frac{\Pi^{new}_{\gamma\gamma}(m_Z^2)}{m_Z^2}\right]$$

$$T \equiv \frac{1}{\alpha_e}\left[\frac{\Pi^{new}_{WW}(0)}{m_W^2} - \frac{\Pi^{new}_{ZZ}(0)}{m_Z^2}\right]$$

$$U \equiv \frac{4s^2}{\alpha_e}\left[\frac{\Pi^{new}_{WW}(m_W^2)-\Pi^{new}_{WW}(0)}{m_W^2} - \frac{c}{s}\frac{\Pi^{new}_{Z\gamma}(m_Z^2)}{m_Z^2} - \frac{\Pi^{new}_{\gamma\gamma}(m_Z^2)}{m_Z^2}\right] - S$$

$c, s = c_\theta, s_\theta$, $\alpha_e = \frac{e^2}{4\pi}$, Π^{new} = only new particles

$S = T = U = 0$ in SM.

Note: Always evaluate at m^2_{pole} or 0.

Easiest to explain T:

$\Pi_{WW}(0)$ = mass^2 of W (location of pole)

$\Pi_{ZZ}(0)$ = mass^2 of Z

T evaluates corrections to m_W^2 + m_Z^2 from new physics. Looking for a violation of custodial symmetry.

Tree-level relation: $m_W^2 = m_Z^2 \cdot c_W^2$

From new physics, $\tilde{m}_W^2 = m_W^2 + \delta m_W^2$, $\tilde{m}_Z^2 = m_Z^2 + \delta m_Z^2$, + test $\delta^2_{mW} = \delta m_Z^2 \cdot c_W^2$. (or $c_W^2 + \delta c_W^2$).

Input parameters + derived params.

S evaluates WFR at m_Z vs. WFR at $m=0$.

Essentially, how Z-like is photon @ m_Z +

Essentially, how Z-like is photon @ m_Z + how photon-like is Z @ $m=0$.

Modern language:

pp collider $\Rightarrow$ can't control $\sqrt{s}$ much $\Rightarrow$ instead study full Q^2 distributions in EW physics.

Natural language dim-6 SM EFT.

1-generation 59 operators. (76 operators w/ $\cancel{B}$)

3-generations 2499 ops.

$$O_S = \frac{1}{\Lambda^2} H^\dagger \sigma^i H W^i_{\mu\nu} B^{\mu\nu}$$

$$O_T = \frac{1}{\Lambda^2} |H^\dagger D_\mu H|^2$$

broken phase $H \to \frac{1}{\sqrt{2}}(v+h)$.

still SM $\mathcal{L}$.

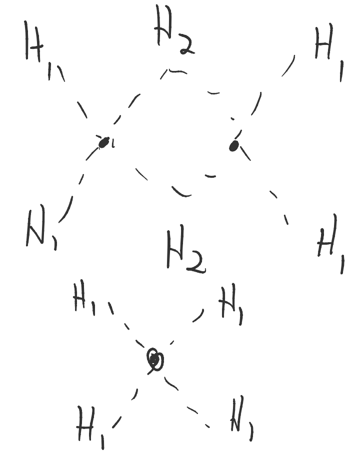
$D_\mu H \ni i g \tau^a \frac{W^a}{\sqrt{2}} H$

$$= \frac{C_T}{\Lambda^2}\left(\frac{v}{2} W_\mu v\right)^2 = \frac{C_T}{\Lambda^2}\frac{(v+h)^4}{4} W^a_\mu W^{b\mu} \frac{\tau^a \tau^b}{2}$$

$$= \frac{C_T g^2 v^4}{16\Lambda^2} W^a_\mu W^{a\mu}$$

SM: $|D_\mu H|^2 \to m_W^2 W^a_\mu W^{a\mu}$

O_T gives $\delta m_W^2 = \frac{g^2 v^4}{16 \Lambda^2} C_T$



EFT: use only light dof. to express effects of heavy dofs.