Lecture 11. (1)

January 6, 2021

QFT III: The Standard Model + EW Theory

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Collider Kinematics + α_s corrections to PDFs / initial + final state radiation + jet formation

Recall the parton model for cross sections of deep inelastic scattering:

$$\sigma(e^-(k)\,p(P) \to e^-(k') + X)$$
$$= \int_0^1 d\xi \sum_f f_f(\xi)\,\sigma(e^-(k)\,q_f(\xi P) \to e^-(k') + q_f(p'))$$

where the quark carries momentum fraction ξ of the original proton. This parton model is accurate at leading order in α_s, but what happens at next to leading order? At NLO, we must include new PDFs + modify cross section

Review kinematics:

Particle 4-momentum (p_x, p_y, p_z, E)

For beam axis along $\hat{z}$,

$$E = m_T \cosh y, \quad p_z = m_T \sinh y, \quad \text{with}$$
$$m_T^2 = m^2 + p_x^2 + p_y^2 = \text{"transverse mass"}$$

Rapidity $y = \frac{1}{2}\ln\left(\frac{E+p_z}{E-p_z}\right) = \ln\left(\frac{E+p_z}{m_T}\right) = \tanh^{-1}\left(\frac{p_z}{E}\right)$

Under boost in longitudinal direction, rapidity differences are <u>invariant</u>.

For $m \ll |\vec{p}|$, $y = \frac{1}{2}\ln\left(\frac{\cos^2(\theta/2) + m^2/4p^2 + \ldots}{\sin^2(\theta/2) + m^2/4p^2 + \ldots}\right)$

$\approx -\ln\tan\left(\frac{\theta}{2}\right) \equiv \eta$, pseudorapidity

for $\cos\theta = \frac{p_z}{|\vec{p}|}$.

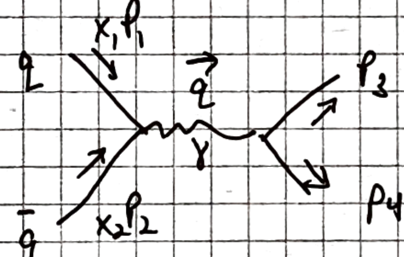
So, we generally use E, $y \approx \eta$, + p_T to characterize collider 4-momenta.

Consider pair production events from $q\bar{q}$, i.e. $q\bar{q} \to e^+e^-$.

The cross section (QED) is

$$\sigma(q\bar{q} \to \ell^+\ell^-) = \frac{1}{3} Q_f^2 \frac{4\pi\alpha^2}{3\hat{s}}.$$

From measuring the ℓ^+ + ℓ^- 4-momenta, we get the virtuality of the Drell-Yan photon: $M^2 = q^2 = (p_3 + p_4)^2$



In pp COM frame, $P_1 = (E, 0, 0, E)$ + $P_2 = (E, 0, 0, -E)$

$$q = x_1 P_1 + x_2 P_2 = ((x_1 + x_2)E, 0, 0, (x_1 - x_2)E)$$

$$M^2 = x_1 x_2 4E^2 = x_1 x_2 s.$$

Using $q^0 = M\cosh Y$, $\cosh Y = \frac{(x_1 + x_2)E}{2E\sqrt{x_1 x_2}} = \frac{1}{2}\left(\sqrt{\frac{x_1}{x_2}} + \sqrt{\frac{x_2}{x_1}}\right)$

and this gives $\exp Y = \sqrt{\frac{x_1}{x_2}}$. (Trig. identity)

We can invert + determine x_1 + x_2:

$$x_1 = \frac{M}{\sqrt{s}} e^{Y}, \quad x_2 = \frac{M}{\sqrt{s}} e^{-Y}$$

Consider the cross section integral over x_1 + x_2:

$$\sigma\left(p(P_1)+p(P_2)\to \ell^+\ell^- + X\right)$$

$$= \int_0^1 dx_1 \int_0^1 dx_2 \sum_f f_f(x_1)\, f_{\bar{f}}(x_2) \cdot \hat{\sigma}\left(q_f(x_1 P) + \bar{q}_f(x_2 P) \to \ell^+\ell^-\right)$$

We can change variables from x_1 + x_2 to M^2 + Y:

Jacobian $\dfrac{\partial(M^2, Y)}{\partial(x_1, x_2)} = \begin{vmatrix} x_2 s & x_1 s \\ \frac{1}{2x_1} & \frac{-1}{2x_2} \end{vmatrix} = s = \dfrac{M^2}{x_1 x_2}$

We get the differential xsec:

$$\frac{d^2\sigma}{d(M^2)\,dY}\left(pp\to \ell^+\ell^- + X\right)$$

$$= \sum_f x_1 f_f(x_1)\, x_2 f_{\bar{f}}(x_2)\, \frac{1}{3} Q_f^2\, \frac{4\pi\alpha^2}{3M^4}$$

So the doubly differential Drell-Yan cross section gives granular information about PDFs.

Dijet production at pp colliders.

Simplify to 2-flavor QCD: $u, d, g, \bar{u}, \bar{d}$.

Since jets can form from light quarks + gluons, and since all quarks are constituent partons of the proton, the $pp \to jj$ process has many different partonic processes. Most have a QED equivalent + only need to be modified by the necessary color factor.

1. $ud \to ud$ $\quad [e^-\mu \to e^-\mu]$

The QED is $\dfrac{d\sigma}{d\hat{t}} = \dfrac{2\pi\alpha^2}{\hat{s}^2}\left(\dfrac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}\right)$

The QCD color factor is

$\frac{1}{3}\frac{1}{3}\sum_{i,i'}\sum_{j,j'}\left[(t^a)_{i'i}(t^a)_{j'j}\right]^2$ for avg. over init. + sum of final colors.

$$\frac{1}{3}\frac{1}{3}\, tr[t^b t^a]\, tr[t^b t^a] = \frac{1}{9}[C(r)]^2 \delta^{ab}\delta^{ab} = \frac{1}{9}\cdot\frac{1}{4}\cdot 8 = \frac{2}{9}$$

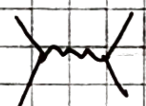
So $$\frac{d\sigma}{d\hat t} = \frac{4\pi\alpha_s^2}{9\hat s^2}\left(\frac{\hat s^2+\hat u^2}{\hat t^2}\right)$$

Crossing symmetry: $u\bar u \to d\bar d$ is s-channel.

$$\frac{d\sigma}{d\hat t}(u\bar u\to d\bar d) = \frac{4\pi\alpha_s^2}{9\hat s^2}\left(\frac{\hat t^2+\hat u^2}{\hat s^2}\right)$$

(like $e^+e^-\to\mu^+\mu^-$)

Now, consider $u\bar u\to u\bar u$: has s-channel & t-channel.

Like $e^+e^-\to e^+e^-$:  + 

$$\frac{d\sigma}{d\hat t} = \frac{4\pi\alpha_s^2}{9\hat s^2}\left(\frac{\hat s^2+\hat u^2}{\hat t^2}+\frac{\hat t^2+\hat u^2}{\hat s^2}-\frac{2\hat u^2}{3\hat s\hat t}\right)$$

Again, crossing gives $uu\to uu$ & $\bar u\bar u\to\bar u\bar u$.

$$\frac{d\sigma}{d\hat t}(uu\to uu) = \frac{4\pi\alpha_s^2}{9\hat s^2}\left(\frac{\hat u^2+\hat s^2}{\hat t^2}+\frac{\hat t^2+\hat s^2}{\hat u^2}-\frac{2}{3}\frac{\hat s^2}{\hat u\hat t}\right)$$

Next, $q\bar q\to gg$, like $e^+e^-\to\gamma\gamma$, but with extra diagram.

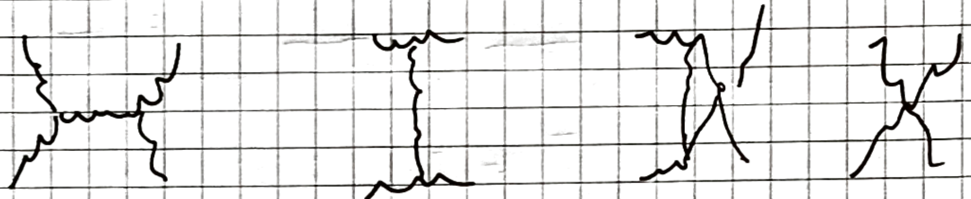
$$\frac{d\sigma}{d\hat t}(q\bar q\to gg) = \frac{32\pi\alpha_s^2}{27\hat s^2}\left(\frac{\hat u}{\hat t}+\frac{\hat t}{\hat u}-\frac{9}{4}\left(\frac{\hat t^2+\hat u^2}{\hat s^2}\right)\right)$$

Cross for $gg\to q\bar q$ & $gq\to gq$ & $g\bar q\to g\bar q$.
& avg. differently.

$$\frac{d\sigma}{d\hat t}(gg\to q\bar q) = \frac{\pi\alpha_s^2}{6\hat s^2}\left(\frac{\hat u}{\hat t}+\frac{\hat t}{\hat u}-\frac{9}{4}\left(\frac{\hat t^2+\hat u^2}{\hat s^2}\right)\right)$$

$$\frac{d\sigma}{d\hat t}(gq\to gq) = \frac{4\pi\alpha_s^2}{9\hat s^2}\left(-\frac{\hat u}{\hat s}-\frac{\hat s}{\hat u}+\frac{9}{4}\left(\frac{\hat s^2+\hat u^2}{\hat t^2}\right)\right)$$

Finally, $gg \to gg$: 4 diagrams, no QED equivalent



$$\frac{d\sigma}{d\hat{t}} = \frac{9\pi\alpha_s^2}{2\hat{s}^2}\left(3 - \frac{\hat{t}\hat{u}}{\hat{s}^2} - \frac{\hat{s}\hat{u}}{\hat{t}^2} - \frac{\hat{s}\hat{t}}{\hat{u}^2}\right)$$

These LO predictions generally work well, given the $\alpha_s(Q^2)$ uncertainty + lack of NLO correction.