08.128.809 Theoretische Elementarteilchenphysik
Quantum Field Theory II

Homework set 6

Due July 8, 2021
Please note how long it took you to solve each problem!

6-1, 35 pts. Basic model building. Consider the Yukawa coupling of an up-quark type with the Standard Model Higgs boson in the broken phase of electroweak symmetry.

$$\mathcal{L} = -\frac{y_u}{\sqrt{2}} h \bar{u} u \ . \tag{1}$$

In this exercise, we will focus on using the field content of a one generation Standard Model, namely the SM fermions $u \sim (3, 2/3)$, $d \sim (3, -1/3)$, $\nu \sim (1, 0,$ and $e \sim (1, -1)$, as well as the Higgs scalar field, $h \sim (1, 0)$. Here, the first entry is the color representation (either triplet or singlet) and the second entry is the EM charge.

A, 5 pts. Draw the tree-level Yukawa coupling diagram for the up quark to the Higgs boson and give the Feynman rule.

B, 5 pts. Draw the two possible 1PI irreducible one-loop corrections topologies to the $h - \bar{u} - u$ coupling. By topology, we mean that the quantum numbers (i.e. color and EM charges) of the virtual particles mediating the loop are kept undetermined, and only the restrictions of spin conservation and renormalizable tree-level couplings are imposed.

C, 5 pts. For the two topologies in part B, label the internal lines assuming the internal fermion is the up quark and the scalar is the Higgs boson. What renormalizable Lagrangian term do you have to add to Eq. (1) and where does this term originate in the Standard Model Lagrangian?

D, 20 pts. The key point about part C is that once one of the internal lines is fixed, the quantum numbers for the other internal particle is also determined up to a modular factor. Restricting to only *new* scalars and the SM fermions u, d, ν, and e as intermediate particles, specify the quantum numbers (color and EM charges) of the scalar fields needed to complete the 1PI-irreducible loops. To avoid repetition, you should take a colored representation for the scalar in the u case, but you can take a colorless representation for the scalar in the d case.

Extra credit, E, 20 pts. Replace the original external Higgs scalar by a gluon (the photon case is very similar), and also allow the possibility that the gluon propagates inside the loop, so we are now considering 1-loop corrections to the up quark coupling to the gluon. Taking the case where the internal fermion is the down quark, and using the same scalar field you found in part D for the down quark case, draw the 1PI irreducible and the 1PI reducible (external leg corrections) in order to get an IR-finite result. Contrast this with the case when the internal fermion is the electron. From the quantum numbers between the two cases, are you guaranteed

that the group theory factor is equivalent? Finally, consider the case of the intermediate down quark again, but allow the scalar field to be colored. What new diagram can you now draw, and does the group theory calculation still match the colorless scalar version? The goal of this exercise is to demonstrate how group theory factors factorize from the spacetime loop structure.

6-2, 50 pts. Practical Standard Model knowledge / trivia. The following are simple facts about the SM that you should know.

A, 5 pts. What Higgs decay modes were initially observed in the 2012 discovery announcement by ATLAS and CMS?

B, 15 pts. What are the five leading production modes of the Higgs boson in the Standard Model? Draw the Feynman diagram for each process.

C, 10 pts. What are the masses of the Higgs boson, Z boson, $W^{\pm}$ boson, and top and bottom quarks?

D, 6 pts. What are the masses of the electron, muon, and tau?

E, 2 pts. What is the dominant decay mode of the top quark?

F, 4 pts. What physical observable is measured to extract the Fermi constant G_F? What SM degree of freedom is integrated out to generate the Fermi interaction (4-fermion, dim-6 operator)?

G, 2 pts. What is the dominant source of mass for the proton and the neutron?

H, 2 pts. Which $b\bar{b}$ resonance was dominantly produced at BaBar and Belle in order to study b-quark properties?

I, 4 pts. What is the largest mixing angle in the CKM matrix, *i.e.* what angle provides the leading deviation from the identity matrix? What is the value of the angle?