

08.128.809 Theoretische Elementarteilchenphysik

Quantum Field Theory II

Homework set 1

Due April 23, 2021; e-mail (photo or scan) to yu001@uni-mainz by start of discussion session

Please note how long it took you to solve each problem

1-1, 15 pts. (A) For quantum field theories in $D = 2$ (1+1), $D = 3$ (2+1), and $D = 6$ (5+1) dimensions (# spatial + # time dimensions), give the scaling dimension of scalar fields, fermions, and vector fields. (B) Characterize the following Lagrangian terms as super-renormalizable, renormalizable, or non-renormalizable in each $D = 2, 3$, and 6: cubic scalar interaction $\lambda\phi^3$, Yukawa coupling $y\phi\bar{\psi}\psi$, and QED gauge coupling in $D = 2, 3$, and 6 dimensions. For each interaction, specify the scaling dimension of the corresponding coupling constant. (Starting hint: recall that Lagrangian kinetic terms always involve two spacetime derivatives for bosons and one spacetime derivative for fermions.)

1-2, 25 pts Demonstrate Furry's theorem (see problem 10.1 in Peskin and Schroeder) that the one-loop matrix element for three external photons in QED vanishes.

1-3, 35 pts Problem 10.2 of Peskin and Schroeder. Consider the pseudoscalar Yukawa Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}m^2\phi^2 + \bar{\psi}(i\not{\partial} - M)\psi - iy\phi\bar{\psi}\gamma^5\psi, \quad (1)$$

where ϕ is a real scalar field and ψ is a Dirac fermion. (A) Determine the superficially divergent amplitudes and the Feynman rules for renormalized perturbation theory. Specify the necessary counterterms. You should find a superficially divergent 4ϕ amplitude, which requires a Lagrangian term,

$$\delta\mathcal{L} = \frac{\lambda}{4}\phi^4$$

and a matching counterterm. Are other interactions required? (B) Compute the pole as $D \rightarrow 4$ of each counterterm to one-loop order and specify the renormalization conditions.

1-4, 25 pts Model with multiple $U(1)$ gauge symmetries. (A) Write the most general, renormalizable, gauge-invariant Lagrangian for a $U(1) \times U(1)$ gauge symmetry theory with

the following fields and charges:

Fields and gauge symmetry charges	$U(1)_1$	$U(1)_2$
ϕ_1 (Complex scalar field)	1	0
ϕ_2 (Complex scalar field)	1	-1
ϕ_3 (Complex scalar field)	0	2
ψ_1 (Fermion)	1/2	0
ψ_2 (Fermion)	0	-1
ψ_3 (Fermion)	1/2	1/2
ψ_4 (Fermion)	3/2	-1

The Lagrangian should include all appropriate covariant kinetic terms, mass terms, and possible non-gauge interactions. The Lagrangian should also include the gauge fields for the two $U(1)$ symmetries (you can call them A_μ and B_μ , respectively). (B) Draw all the possible 1-loop contributions for the vacuum polarization diagrams of the two $U(1)$ gauge fields. Note that this includes the diagonal A_μ - A_ν and B_μ - B_ν wavefunction renormalizations as well as the mixed A_μ - B_ν loop.