08.128.809 Theoretische Elementarteilchenphysik
Quantum Field Theory II

Homework set 4

Due June 10, 2021
Please note how long it took you to solve each problem!

4-1, 35 pts. Flavor physics is the study of global symmetries and their breaking patterns via interactions. Most commonly, fields are redefined to have *canonical kinetic terms* and *diagonal mass terms*, leaving the interactions, which involve three fields or more, in the most general form. Consider the Lagrangian

$$\mathcal{L} = \bar{\psi}_i(i\not{D})\psi_i - m_{ij}\bar{\psi}_i\psi_j - \left(y_{ij}\phi\overline{\psi}_i\psi_j + \text{ h.c.}\right) \tag{1}$$

where the i and j indices run from 1 to N_F. Assume $\not{D}$ is the same for all of the ψ fields and ϕ is a real scalar field..

A, 5 pts. What is the largest symmetry transformation of ψ that leaves $\bar{\psi}_i(i\not{D})\psi_i$ invariant?

B, 5 pts. What is the restriction on the matrix m_{ij} such that the Lagrangian is Hermitian? (Recall that Lagrangians must be Hermitian in order to satisfy CPT-invariance.) Tip: it is easiest to add the Hermitian conjugate and then manipulate the terms to have the same field structure. Given the global symmetry transformation in part A, can m_{ij} be simplified to always have diagonal form?

C, 10 pts. The Yukawa matrix is generally independent of the mass matrix. What are the restrictions on y_{ij} such that the Lagrangian is Hermitian? And what does the symmetry transformation performed in part B do to the y_{ij} matrix?

D, 15 pts. In the Standard Model, the basic building block of fermion fields are Weyl fermions, which are chiral representations of fermions. (This is in contrast with the fermions in Eq. (1), which are 4-component Dirac fermions.) Recognizing that 2-component Weyl fermions are left-handed or right-handed projections of Dirac fermions, we can write the quark Lagrangian as (with minor simplifications)

$$\begin{aligned}\mathcal{L} = &\bar{u}_{L,i}(i\not{D})u_{L,i} + \bar{d}_{L,i}(i\not{D})d_{L,i} + \bar{u}_{R,i}(i\not{D})u_{R,i} + \bar{d}_{R,i}(i\not{D})d_{R,i} \\ &- \left(y^u_{ij}\left(\frac{v+h}{\sqrt{2}}\right)(\bar{u}_{L,i}u_{R,j}) + y^d_{ij}\left(\frac{v+h}{\sqrt{2}}\right)\left(\bar{d}_{L,i}d_{R,j}\right) + \text{ h.c.}\right) ,\end{aligned} \tag{2}$$

where the i, j flavor indices range from 1 to 3. Assuming the four Weyl fields can be rotated independently, what is the global symmetry of this Lagrangian if all y^u_{ij} and y^d_{ij} are neglected? Using this flavor symmetry, demonstrate that a basis rotation exists where the fermion fields, in this new basis, experience diagonal Yukawa interactions. Write the corresponding mass terms and Yukawa interactions in this new basis using Dirac fermions. *Remark: In the SM, this basis choice is only violated by the charged $SU(2)$ interactions in the $i\not{D}$ terms of the u_L and d_L fields, giving rise to the Cabibbo-Kobayashi-Maskawa matrix.*

4-2, 20 pts. Electron electric dipole moment. Given the Lagrangian

$$
\begin{aligned}
\mathcal{L} &= \bar{e}(i\not{D} - m_e)e - (\frac{y_e}{\sqrt{2}}\bar{e}_L h e_R + \text{h.c.}) \\
&+ \bar{t}(i\not{D} - m_t)t - (\frac{y_t}{\sqrt{2}}\bar{t}_L h t_R + \text{h.c.}) \\
&- \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}Z_{\mu\nu}Z^{\mu\nu} + \frac{1}{2}m_Z^2 Z^\mu Z_\mu + \frac{1}{2}(\partial_\mu h)^2 - \frac{1}{2}m_h^2 h^2
\end{aligned} \tag{3}
$$

where

$$
\begin{aligned}
D_\mu e &\equiv \left(\partial_\mu + ieA_\mu - i\frac{g}{4\cos\theta_w}Z_\mu\left((-1 + 4\sin^2\theta_w) + \gamma_5\right)\right)e \\
D_\mu t &\equiv \left(\partial_\mu - i\frac{2}{3}eA_\mu - i\frac{g}{4\cos\theta_w}Z_\mu\left(\left(1 - \frac{8}{3}\sin^2\theta_w\right) - \gamma_5\right)\right)t
\end{aligned} \tag{4}
$$

and

$$
F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu \ , Z_{\mu\nu} \equiv \partial_\mu Z_\nu - \partial_\nu Z_\mu \ , \tag{5}
$$

draw all 1PI diagrams for the electron coupling to a photon at tree, one-loop, and two-loop level. Nomenclature: in this Lagrangian, the Higgs field is h, e is the electron, t is the top quark, A_μ is the photon, and Z_μ is the Z-boson. *Remark: In the Standard Model, the EDM of the electron is expected to be tiny (smaller than $10^{-38}e$ cm), and the leading contribution in the limit of massless neutrinos arises from 4-loop diagrams. The current experimental sensitivity is at the level of $10^{-29}e$ cm, given by the ACME collaboration. The two-loop diagrams with internal h and Z propagators are called "Barr-Zee" diagrams, which provide the leading constraint on possible CP-violating phase of the top quark Yukawa coupling.*

4-3, 45 pts. In beyond the Standard Model physics, an extension of the Standard Model QCD gauge symmetry is natural to consider. In particular, the SM $SU(3)_c$ color symmetry is enhanced to a product group of $SU(3)_1 \times SU(3)_2$ at a high scale, and the diagonal subgroup [where the separate generators of each $SU(3)$ are identified with each other] is the familiar $SU(3)_c$ symmetry of the Standard Model. The massless gluon, which mediates the unbroken $SU(3)_c$ symmetry, then acquires a partner X, which is a massive color octet vector (commonly referred to as an axigluon or coloron). A relevant set of Lagrangian terms is then

$$
\mathcal{L} = \bar{t}(i\not{\partial} - m_t)t + g_s(\lambda\bar{t}_L\gamma_\mu t^a X^{a,\mu}t_L + \kappa\bar{t}_R\gamma_\mu t^a X^{a,\mu}t_R) - \frac{1}{4}X^a_{\mu\nu}X^{a,\mu\nu} + \frac{1}{2}m_X^2 X_\mu X^\mu \ . \tag{6}
$$

A, 15 pts. Rewrite the Lagrangian coupling for X_μ to t by using $t_L = P_L t$ and $t_R = P_R t$. What is the Feynman rule for X_μ interacting with t?

B, 20 pts. Calculate the decay width for $X \to t\bar{t}$. Here, t transforms in the fundamental representation of the $SU(3)_c$ symmetry and X transforms in the adjoint representation. When evaluating the polarization sum of the matrix element squared, you will need to use

$$\sum_{\text{polarizations}} \epsilon_\mu \epsilon^*_\nu = (-g_{\mu\nu} + \frac{k_\mu k_\nu}{m_X^2}) \;, \tag{7}$$

where k is the momentum of the X particle. (This is the generic form of the modification of the polarization sum when the external vector field is massive. We will discuss this further in the context of the Higgs mechanism and Goldstone boson equivalence, but for background reading, see sections 21.1 and 21.2 of Peskin and Schroeder.)

C, 5 pts. What is the partial width for $X \to t\bar{t}$ if $m_t \to 0$?

D, 5 pts. Assuming the only decay channel is $X \to t\bar{t}$, and for $m_X = 1$ TeV, $m_t = 173$ GeV, $g_s\lambda = g_s\kappa = 0.1$, what is the lifetime of X?