Lecture 16.

QFT II.

Felix Yu

July 1, 2020.

Last time: Goldstone's theorem + SSB in EW theory.

Today: Higgs mechanism in SM

How does Higgs break EW symmetry + give mass to $W^{+/-}$ + Z?

$H \sim (1, 2, 1/2)$

$\mathcal{L}_{Higgs} \supset |D_\mu H|^2 - \mu^2 |H|^2 - \lambda |H|^4, \quad \mu^2 < 0$

$$D_\mu H = \partial_\mu H - ig\tau^i W^i_\mu H - \frac{ig'}{2} B_\mu H$$

For $\langle H \rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}$ by global SU(2) rotation,

$$\tau^i = \frac{\sigma^i}{2}, \; v^2 = \frac{-\mu^2}{\lambda}$$

$$D_\mu H \supset -ig\frac{\tau^i}{\sqrt{2}} W^i_\mu \begin{pmatrix} 0 \\ v \end{pmatrix} - \frac{ig'}{2\sqrt{2}} B_\mu \begin{pmatrix} 0 \\ v \end{pmatrix}$$

Use
$$\tau^i W^i_\mu = \frac{1}{2}\left(\sigma^1 W^1_\mu + \sigma^2 W^2_\mu + \sigma^3 W^3_\mu\right) = \frac{1}{2}\begin{pmatrix} W^3_\mu & W^1_\mu - iW^2_\mu \\ W^1_\mu + iW^2_\mu & -W^3_\mu \end{pmatrix}$$

Get
$$D_\mu H \supset \begin{pmatrix} \frac{1}{2\sqrt{2}}(-ig)(W^1_\mu - iW^2_\mu)v \\ \frac{1}{2\sqrt{2}}(-ig)(-W^3_\mu)v - \frac{ig'}{2\sqrt{2}} B_\mu v \end{pmatrix}$$

So
$$|D_\mu H|^2 \Big|_{v^2} = \frac{1}{8}\left(g^2 v^2 (W^1_\mu - iW^2_\mu)(W^1_\mu + iW^2_\mu)\right) + \frac{1}{8} v^2 \left(-gW^3_\mu + g' B_\mu\right)^2$$

Define $W^+_\mu = \frac{1}{\sqrt{2}}(W^1_\mu - iW^2_\mu)$, $W^-_\mu = \frac{1}{\sqrt{2}}(W^1_\mu + iW^2_\mu)$

[in analogy with raising + lowering operators for spin 1/2 in QM]

Also, identify mass eigenstates:

$$Z_\mu = \frac{1}{\sqrt{g^2+g'^2}}\left(-g W^3_\mu + g' B_\mu\right)$$

$$A_\mu = \frac{1}{\sqrt{g^2+g'^2}}\left(g' W^3_\mu + g B_\mu\right)$$

Normalizations ensure kinetic terms of fields are canonically normalized.

We get $|D_\mu H|^2\Big|_{v^2} = \frac{1}{4} g^2 v^2 W^+ W^- + \frac{(g^2+g'^2)}{8} v^2 Z_\mu Z^\mu$

giving $m_W^2 = \frac{g^2 v^2}{4}$, $\quad m_Z^2 = \frac{(g^2+g'^2)^2}{4}$

(complex vector) (real vector)

For arbitrary $D_\mu = \partial_\mu - i g W^i_\mu \tau^i - i g' Y B_\mu$, we can find the interactions with the $W^{+/-}, Z, A$ mass eigenstates as:

$$D_\mu = \partial_\mu - \frac{ig}{\sqrt{2}}\left(W^+_\mu T^+ + W^-_\mu T^-\right) - \frac{i Z_\mu}{\sqrt{g^2+g'^2}}\left(g^2 T^3 - g'^2 Y\right)$$

$T^\pm = T^1 \pm i T^2 = \sigma^\pm$

$$- \frac{i g g'}{\sqrt{g^2+g'^2}} A_\mu \left(T^3 + Y\right)$$

So coupling to $W^{+/-}$: $\frac{g}{\sqrt{2}}$.

Z: $\dfrac{g^2 T^3 - g'^2 Y}{\sqrt{g^2+g'^2}}$

A: $(T^3+Y)\dfrac{g g'}{\sqrt{g^2+g'^2}} \equiv Q e$ for

$Q \equiv T^3 + Y$ + $e \equiv \dfrac{g g'}{\sqrt{g^2+g'^2}}$, QED coupling.

Define $c_W = \dfrac{g}{\sqrt{g^2+g'^2}}$, $s_W = \dfrac{g'}{\sqrt{g^2+g'^2}}$, $g = \dfrac{e}{s_W}$

(weak mixing angle).

$$\Rightarrow D_\mu = \partial_\mu - \frac{ig}{\sqrt{2}}\left(W^+_\mu T^+ + W^-_\mu T^-\right) - \frac{ig}{c_W} Z_\mu \left(T^3 - s_W^2 Q\right) - i e A_\mu Q$$

Higgs vev also gives masses for quarks and ~~c~~ charged leptons.

$$\mathcal{L} = -y_u \bar{Q}_L \tilde{H} u_R - y_d \bar{Q}_L H d_R - y_e \bar{L}_L H e_R + h.c.$$

Each Yukawa matrix is 3×3 in flavor space.

SU(2) indices contract on $\bar{Q}_L \tilde{H}$, $\bar{Q}_L H$, $\bar{L}_L H$, respectively.

$i\sigma_2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$, so $\tilde{H} \equiv i\sigma_2 H^* = \frac{1}{\sqrt{2}} \begin{pmatrix} v + h^0 - iG_0 \\ -G^- \end{pmatrix}$

Get $\mathcal{L} \supset -\frac{y_u^{ij}}{\sqrt{2}} (v + h_0) \bar{u}_{Li} u_{Rj} - \frac{y_d^{ij}}{\sqrt{2}} (v + h_0) \bar{d}_L^i d_R^j$

$$- \frac{y_e^{ij}}{\sqrt{2}} (v + h_0) \bar{e}_{Li} e_{Rj} + h.c.$$

Can demonstrate that $U(3) = U(1) \times SU(3)$ global symmetries can be used to rotate to mass eigenstate basis + diagonalize y_u, y_d, y_e.

$$\mathcal{L} \supset \bar{u}_L U_{u_L}^\dagger U_{u_L} y_u U_{u_R}^\dagger U_{u_R} u_R \frac{1}{\sqrt{2}} (v + h)$$

$$+ \bar{d}_L U_{d_L}^\dagger U_{d_L} y_d U_{d_R}^\dagger U_{d_R} d_R \frac{1}{\sqrt{2}} (v + h)$$

$$+ \bar{e}_L U_{e_L}^\dagger U_{e_L} y_e U_{e_R}^\dagger U_{e_R} e_R \frac{1}{\sqrt{2}} (v + h) + h.c.$$

Choose U's s.t. $U_{u_L} y_u U_{u_R}^\dagger = y_u^{diag}$, etc.

Then, $U_{u_R} u_R = u_R^{mass}$, $U_{u_L} u_L = u_L^{mass}$

Then, we get

$$\mathcal{L} \supset \bar{u}_L^{mass} y_u^{diag} u_R^{mass} \frac{(v+h)}{\sqrt{2}} + h.c. + \ldots$$

$$= \bar{u}^m u^m \left(\frac{v+h}{\sqrt{2}}\right) y_u^{diag} + (\text{down-type + charged leptons})$$

So $\frac{y_u^{diag} v}{\sqrt{2}} = m_u$ mass terms, all real.

Performing the same rotation to mass eigenstate basis in the kinetic terms for the fermions keeps the Z + γ terms unchanged:

$$\bar{u}_L U_{u_L}^\dagger U_{u_L} (\#) Z_\mu \gamma^\mu U_{u_L}^\dagger U_{u_L} u_L$$

$$= \bar{u}_L^{mass} U_{u_L} (\#) \not{Z} U_{u_L}^\dagger u_L^{mass}$$

$$= \bar{u}_L^{mass} (\#) \not{Z} u_L^{mass}$$

The $W^{+/-}$ interactions are not diagonal, however in the quark mass basis.

$$\bar{Q}_L i \not{D} Q_L \supset (\bar{u}_L \ \bar{d}_L) \, i \left(\frac{g}{\sqrt{2}} W_\mu^+ \sigma^+ \gamma^\mu + \frac{g}{\sqrt{2}} W_\mu^- \sigma^- \gamma^\mu \right) \begin{pmatrix} u_L \\ d_L \end{pmatrix}$$

$$\Rightarrow \bar{u}_L U_{u_L}^\dagger U_{u_L} W_\mu^+ \frac{g}{\sqrt{2}} \gamma^\mu U_{d_L}^\dagger U_{d_L} d_L$$

$$= \bar{u}_L^{mass} U_{u_L} U_{d_L}^\dagger W_\mu^+ \frac{g}{\sqrt{2}} d_L^{mass}$$

$$\text{and } \bar{d}_L^{mass} (U_{d_L} U_{u_L}^\dagger) W_\mu^- \frac{g}{\sqrt{2}} \gamma^\mu u_L^{mass}$$

The particular combination $(U_{u_L} U_{d_L}^\dagger)$ signifies the mismatch between gauge + mass eigenstates for quarks in charged current interactions $\Rightarrow$ $V_{CKM} = U_{u_L} U_{d_L}^\dagger$

$$= \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Cabibbo-Kobayashi-Maskawa

3 mixing angles, 1 $\not{CP}$ phase

Since charged currents are not diagonal, all flavor violation in SM is proportional to V_{CKM}.

On the other hand, Z + Higgs interactions are flavor-conserving at tree-level.

"No tree-level FCNCs"

For neutrinos, have an ~~ang~~ analogous situation since neutrinos are massive $\Rightarrow$ PMNS matrix

Pontecorvo - Maki - Nagawa - Sakata

Just like quarks, PMNS matrix gives mismatch between mass + gauge eigenstates for neutrinos in charged current interactions

For reference, V_{CKM} is nearly identity, while V_{PMNS} is more equal / mixed.

$$\text{PDG} \quad V_{CKM} \simeq \begin{pmatrix} 0.97446 \pm 10^{-4} & 0.22452 \pm 4.4\cdot 10^{-4} & 0.00365 \pm 1.2\cdot 10^{-4} \\ 0.22438 \pm 4.4\cdot 10^{-4} & 0.97359 \pm^{10^{-4}}_{1.1\cdot 10^{-4}} & 0.04214 \pm 7.6\cdot 10^{-4} \\ 0.00896 \pm^{2.4\cdot 10^{-4}}_{2.3\cdot 10^{-4}} & 0.04133 \pm 7.4\cdot 10^{-4} & 0.999105 \pm 3.2\cdot 10^{-5} \end{pmatrix}$$

Offshoot topics: (Many covered in QFT3)

- Dark matter
- Neutrino masses + mixing
- Baryogenesis / Leptogenesis
- S, T, U - Oblique corrections + EW precision
- Top physics
- bottom physics
- Tau physics
- Higgs physics
- Extensions of SM gauge symmetry
- Flavor physics + EFTs, EW Hamiltonian
- Gauge hierarchy problem.