08.128.809 Theoretische Elementarteilchenphysik
Quantum Field Theory II

Homework set 5

Due June 29, 2020
Please note how long it took you to solve each problem!

5-1, 50 pts. Z boson decay widths. The Z boson is the heavy cousin of the electromagnetic photon that arises from spontaneous breaking of the $SU(2)_L \times U(1)_Y$ electroweak symmetry by the Higgs field. In this problem, we will calculate the partial widths of the Z boson to Standard Model fermions. You can assume that the fermion masses are all negligible compared to the Z boson mass.

We start with the expression for the Z current (reproduced from Eqs. 20.79 and 20.80 in Peskin and Schroeder):

$$\mathcal{L} = \frac{1}{2} m_Z^2 Z_\mu Z^\mu + g Z_\mu^0 J_Z^\mu \tag{1}$$

$$\begin{aligned} J_Z^\mu = \frac{1}{\cos\theta_W} \Big[& \bar{\nu}_L \gamma^\mu \frac{1}{2} \nu_L + \bar{e}_L \gamma^\mu (-\frac{1}{2} + \sin^2\theta_W) e_L + \bar{e}_R \gamma^\mu (\sin^2\theta_W) e_R \\ & + \bar{u}_L \gamma^\mu (\frac{1}{2} - \frac{2}{3}\sin^2\theta_W) u_L + \bar{u}_R \gamma^\mu (-\frac{2}{3}\sin^2\theta_W) u_R \\ & + \bar{d}_L \gamma^\mu (-\frac{1}{2} + \frac{1}{3}\sin^2\theta_W) d_L + \bar{d}_R \gamma^\mu (\frac{1}{3}\sin^2\theta_W) d_R \Big] \ . \end{aligned} \tag{2}$$

For the decay width calculations, it is more convenient to re-express the Lagrangian and Feynman rules using 4-component Dirac spinors with vector and axial-vector couplings. To combine the Weyl spinors with different chiralities, recall that $f_L = P_L f$ and $f_R = P_R f$.

A, 5 pts. Rewrite the $g J_Z^\mu$ current as $\bar{f}\left(g_f^V \gamma^\mu + g_f^{AV} \gamma^\mu \gamma^5\right) f$ for $f = \nu$, e, u, and d, and identify the appropriate vector coupling g_f^V and axial-vector coupling g_f^{AV} for each type of fermion.

B, 5 pts. From part (A), write the tree-level Feynman rules for the Z couplings to neutrinos, charged leptons, up-type quarks, and down-type quarks.

C, 20 pts. Calculate the two-body decay width of the Z boson decaying to a massless fermion with the generic coupling structure above: $\bar{f}\left(g_f^V \gamma^\mu + g_f^{AV} \gamma^\mu \gamma^5\right)$. You will need to use the polarization sum for massive vector bosons, but as a reminder, you can neglect the fermion masses.

D, 15 pts. Using part (B), find the explicit two-body decay width for $Z \to \nu\bar{\nu}$, $Z \to e\bar{e}$, and $Z \to u\bar{u}$, $Z \to d\bar{d}$, assuming one flavor of each fermion. Since the quarks carry color charge (and we have suppressed color indices), the sum over final states in $Z \to u\bar{u}$ and $Z \to d\bar{d}$ should include a factor of 3.

E, 5 pts. Since $m_Z = 91.2$ GeV and $m_t = 173.4$ GeV, the Z boson cannot decay into two top quarks, but all the other possible decays to SM fermions are kinematically allowed. Given that we have three generations of each type of fermion (and removing one multiplicity for up-type quarks because of kinematics), what is the total Z decay width, assuming the two-body decays dominate? And what is the Z proper lifetime?

5-2, 50 pts. Perturbative unitarity of electroweak scattering in the Standard Model and demonstration of Goldstone boson equivalence. The major achievement of the ATLAS and CMS experiments at the Large Hadron Collider in CERN was the discovery of the Higgs boson in 2014. One of the strongest and central motivations for the Higgs boson was the theoretical requirement of perturbative unitarity in massive vector boson scattering.

A, 15 pts. The original paper discussing perturbative unitarity in vector boson scattering is by B. Lee, C. Quigg, and H. Thacker, "Weak Interactions at Very High-Energies: The Role of the Higgs Boson Mass," PRD 16 (1977) 1519, which is available on Reader. After reading the paper, in your own words, present the theoretical motivation for a Higgs boson discovery in the electroweak-TeV energy range.

B, 10 pts. Given the Lagrangian of electroweak gauge interactions,

$$
\begin{aligned}
\mathcal{L} \supset\ & \frac{1}{2}(\partial_\mu h)^2 - \frac{1}{2}m_h^2 h^2 + 2\frac{m_W^2}{v} h W_\mu^+ W^{\mu,-} + \frac{m_Z^2}{v} h Z_\mu Z^\mu \\
& - \frac{1}{4} F_{\mu\nu}F^{\mu\nu} \\
& - \frac{1}{2} W_{\mu\nu}^+ W^{-\ \mu\nu} + m_W^2 W_\mu^+ W^{-\ \mu} \\
& - \frac{1}{4} Z_{\mu\nu} Z^{\mu\nu} + \frac{1}{2} m_Z^2 Z^\mu Z_\mu \\
& + ie\cot\theta_w (Z_{\mu\nu} W_\mu^+ W_\nu^- - W_{\mu\nu}^+ Z_\mu W_\nu^- + W_{\mu\nu}^- Z_\mu W_\nu^+) \\
& + ie(F_{\mu\nu} W_\mu^+ W_\nu^- - W_{\mu\nu}^+ A_\mu W_\nu^- + W_{\mu\nu}^- A_\mu W_\nu^+) \\
& + \frac{1}{2}\frac{e^2}{\sin^2\theta_w}(W_\mu^+ W_\mu^+ W_\nu^- W_\nu^- - W_\mu^+ W_\mu^- W_\nu^+ W_\nu^-),
\end{aligned} \tag{3}
$$

where we have adopted unitary gauge, we will study the tree-level diagrams involved in perturbative unitarity. Draw all Feynman diagrams for tree-level scattering of $W^+W^- \to W^+W^-$ and $W^+W^+ \to W^+W^+$.

C, 15 pts. In unitary gauge , the gauge boson propagator for a massive vector is $\frac{-i}{q^2 - m^2}(g^{\mu\nu} - \frac{q^\mu q^\nu}{m^2})$. Write out the matrix elements for $W^+W^+ \to W^+W^+$. The Feynman rules for the $W - W - Z$ and $W - W - A$ couplings are given in Peskin and Schroeder, Figure 21.9 on page 753, and the Feynman rule for the $W - W - h$ coupling is given in Peskin and Schroeder, Figure 20.6 on page 716.

D, 10 pts. Adopting R_ξ gauge with $\xi = 1$ (Feynman gauge), what is the expression for the massive gauge propagator? And what are the new diagrams for $W^+W^+ \to$

W^+W^+ scattering? Comparing between (B) and (C), you can see diagramatically the equivalence between the *longitudinal* modes of the vector boson propagator and the *Goldstone* modes that appear in Feynman gauge.

E, Extra credit, 10 pts. If you sum the matrix elements for $W^+W^+ \to W^+W^+$ but ignore the Higgs contribution, show that the leading E^4 contribution in the matrix element cancels because of gauge coupling equivalence between 3-pt. and 4-pt. vertices. Note there is still a growth in the matrix element at large energies that scales quadratically with energy. A guide for the calculation is Chang, Cheung, Lu, Yuan, "WW Scattering in the Era of Post Higgs Discovery," PRD 87 093005 [arXiv:1303.6335], which is available on Reader.

F, Extra credit, 10 pts. Show that the quadratic energy growth in part (E) is cancelled by the Higgs-mediated diagram. *In Lee, Quigg, and Thacker, by keeping the Higgs mass a free parameter and imposing the requirement of partial wave unitarity, the Higgs mass can be bounded to be below about 800 GeV.*