08.128.809 Theoretische Elementarteilchenphysik
Quantum Field Theory II

Homework set 2

Due May 18, 2020
Please note how long it took you to solve each problem!

2-1, 20 pts. Using Feynman rules derived from the Lagrangian:

$$\begin{aligned}\mathcal{L} = &\, i\bar{f}_1 \partial\!\!\!/ f_1 + i\bar{f}_2 \partial\!\!\!/ f_2 + i\bar{f}_3 \partial\!\!\!/ f_3 - m_1 \bar{f}_1 f_1 - m_2 \bar{f}_2 f_2 - m_3 \bar{f}_3 f_3 \\ &- \frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 - \frac{1}{4}(\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+)(\partial_\mu W_\nu^- - \partial_\nu W_\mu^-) \\ &+ Q_2 e \bar{f}_2 A\!\!\!/ f_2 + Q_3 e \bar{f}_3 A\!\!\!/ f_3 + g(\bar{f}_3 W\!\!\!\!/^{\,+} P_L f_1 + \text{ h.c.})\ ,\end{aligned}$$

write the matrix elements corresponding to the diagrams in Fig. 1. Note: You do not need to evaluate any of the matrix elements. Also, because there is no mass term for either vector, you can use the massless vector propagator from QED.

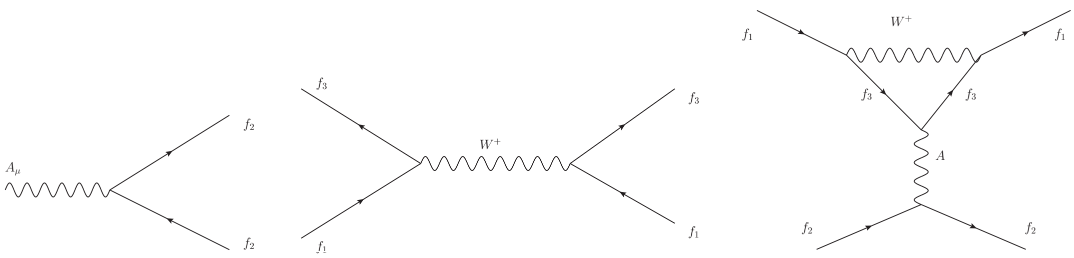

Figure 1: (A.) Vector coupling to two fermions, $A^\mu \to f_2 \bar{f}_2$. (B.) Charged current scattering, $f_1 \bar{f}_3 \to f_1 \bar{f}_3$ via exchange of W^+. (C.) The penguin diagram. You can think of the penguin diagram as a one-loop vertex correction to a vector current where the external vector is allowed to propagate off-shell.

2-2, 25 pts. Using Feynman rules derived from the Lagrangian:

$$\begin{aligned}\mathcal{L} = &\, i\bar{f}_1 \partial\!\!\!/ f_1 + i\bar{f}_2 \partial\!\!\!/ f_2 + i\bar{f}_3 \partial\!\!\!/ f_3 - m_1 \bar{f}_1 f_1 - m_2 \bar{f}_2 f_2 - m_3 \bar{f}_3 f_3 \\ &- \frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 - \frac{1}{4}(\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+)(\partial_\mu W_\nu^- - \partial_\nu W_\mu^-) \\ &+ Q_2 e \bar{f}_2 A\!\!\!/ f_2 + Q_3 e \bar{f}_3 A\!\!\!/ f_3 + g\cos\theta(\bar{f}_3 W\!\!\!\!/^{\,+} P_L f_1 + \text{ h.c.}) + g\sin\theta(\bar{f}_2 W\!\!\!\!/^{\,+} P_L f_1 + \text{ h.c.})\ ,\end{aligned}$$

draw all the diagrams for $f_2 \bar{f}_3 \to f_2 \bar{f}_3$ scattering at tree-level and one-loop. Be sure to include diagrams that cross internal propagator legs. For example, you should have one diagram, known as the box diagram, as shown here: (This diagram is central to understanding the phenomenology of SM mesons.) For the box diagram in Fig. 2, write the matrix element. Does the loop integral superficially converge or diverge? For

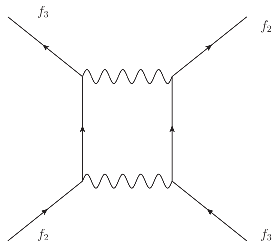

Figure 2: The box diagram.

all of your diagrams, group them according to the dependence on the gauge couplings e and g. Do all diagrams within each class (diagrams that share the same parametric dependence on e and g) share the same superficial degree of divergence? What physical condition ensures sensitivity to UV divergences vanishes? (Extra credit) If you are ambitious, calculate the leading loop-momentum dependence of each diagram within each class and verify that the UV divergence cancels, leaving only a finite correction (you may need to introduce a mass for each gauge boson to regulate IR divergences).

2-3, 30 pts. Using the functional method, derive the Feynman rules in momentum space for the following interaction vertices:

A. $\mathcal{L} = \lambda\phi^4$

B. $\mathcal{L} = g^2\phi^*\phi A_\mu A^\mu$. Note that for the purposes of functional methods, vector fields are simply collections of scalar fields indexed by an extra Lorentz index.

C. $\mathcal{L} = g f_{abc}\partial_\mu A_\nu^a A^{\mu,\ b} A^{\nu,\ c}$. For convenience, define all momenta to flow into the vertex, and assume f_{abc} is totally antisymmetric under interchange of any two [group space] indices.

D, Extra credit, 10 pts. $\mathcal{L} = \frac{1}{4}g^2(f^{eab}A_\mu^a A_\nu^b)(f^{ecd}A^{\mu,\ c}A^{\nu,\ d})$. Again, assume f_{abc} is totally antisymmetric under interchange of any two [group space] indices.

2-4, 25 pts. A. Derive the β-function and the anomalous dimension γ of the fermion field and photon field in QED to leading dependence on the gauge coupling constant e. The calculation is outlined on p. 416 of Peskin and Schroeder.

B. Running couplings in QED. In the Standard Model, the QED coupling is defined by the fine structure constant $\alpha = e^2/(4\pi)$. At low momentum transfer (such as the scale of the electron mass, 511 keV), the fine structure constant is measured to be $\alpha \approx 1/137$. With the beta function of e from part A, what is the scale where QED becomes non-perturbative? You can define this scale roughly by the energy scale when $\alpha \to 1$, which leads to a breakdown in perturbativity.