# 08.128.809 Theoretische Elementarteilchenphysik
# Quantum Field Theory II

**Homework set 1**

**Due May 4, 2020; e-mail (photo or scan) to yu001@uni-mainz by start of discussion session**
**Please note how long it took you to solve each problem**

1-1, 15 pts. (A) For quantum field theories in $D = 2$ (1+1), $D = 3$ (2+1), and $D = 6$ (5+1) dimensions (# spatial + # time dimensions), give the scaling dimension of scalar fields, fermions, and vector fields. (B) Characterize the following Lagrangian terms as super-renormalizable, renormalizable, or non-renormalizable in each $D = 2$, 3, and 6: cubic scalar interaction $\lambda\phi^3$, Yukawa coupling $y\phi\overline{\psi}\psi$, and QED gauge coupling in $D = 2$, 3, and 6 dimensions. For each interaction, specify the scaling dimension of the corresponding coupling constant. (Starting hint: recall that Lagrangian kinetic terms always involve two spacetime derivatives for bosons and one spacetime derivative for fermions.)

1-2, 25 pts Demonstrate Furry's theorem (see problem 10.1 in Peskin and Schroeder) that the one-loop matrix element for three external photons in QED vanishes.

1-3, 35 pts Problem 10.2 of Peskin and Schroeder. Consider the pseudoscalar Yukawa Lagrangian

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu\phi\right)^2 - \frac{1}{2}m^2\phi^2 + \bar{\psi}(i\partial\!\!\!/ - M)\psi - iy\phi\bar{\psi}\gamma^5\psi \ , \tag{1}$$

where $\phi$ is a real scalar field and $\psi$ is a Dirac fermion. (A) Determine the superficially divergent amplitudes and the Feynman rules for renormalized perturbation theory. Specify the necessary counterterms. You should find a superficially divergent $4\phi$ amplitude, which requires a Lagrangian term,

$$\delta\mathcal{L} = \frac{\lambda}{4}\phi^4$$

and a matching counterterm. Are other interactions required? (B) Compute the pole as $D \to 4$ of each counterterm to one-loop order and specify the renormalization conditions.

1-4, 25 pts Model with multiple $U(1)$ gauge symmetries. (A) Write the most general, renormalizeable, gauge-invariant Lagrangian for a $U(1) \times U(1)$ gauge symmetry theory with the following fields and charges:

| Fields and gauge symmetry charges | $U(1)_1$ | $U(1)_2$ |
|---|---|---|
| $\phi_1$ (Complex scalar field) | 1 | 0 |
| $\phi_2$ (Complex scalar field) | 1 | $-1$ |
| $\phi_3$ (Complex scalar field) | 0 | 2 |
| $\psi_1$ (Fermion) | 1/2 | 0 |
| $\psi_2$ (Fermion) | 0 | $-1$ |
| $\psi_3$ (Fermion) | 1/2 | 1/2 |
| $\psi_4$ (Fermion) | 3/2 | $-1$ |

The Lagrangian should include all appropriate covariant kinetic terms, mass terms, and possible non-gauge interactions. The Lagrangian should also include the gauge fields for the two $U(1)$ symmetries (you can call them $A_\mu$ and $B_\mu$, respectively). (B) Draw all the possible 1-loop contributions for the vacuum polarization diagrams of the two $U(1)$ gauge fields. Note that this includes the diagonal $A_\mu$-$A_\nu$ and $B_\mu$-$B_\nu$ wavefunction renormalizations as well as the mixed $A_\mu$-$B_\nu$ loop.